PARAMETRIC SOLUTIONS OF CERTAIN DIOPHANTINE EQUATIONS

By E. T. Bell

1. **Introduction.** The complete integer solution of

$$x_1y_1 + \cdots + x_ny_n = 0 \tag{1.1}$$

is given by the formulas[1]

$$x_i = \alpha\alpha_i, \qquad y_i = -\sum_{j=1}^{i-1} \alpha_j\beta_{j,i} + \sum_{j=1}^{n-i} \alpha_{i+j}\beta_{i,i+j}, \tag{1.2}$$

where the Greek letters denote integer parameters, with the convention (as always) that a summation (or range of values) in which the lower limit exceeds the upper is vacuous. Let $f_i(x_1, \cdots, x_n)$ $(i = 1, \cdots, n)$ be any functions which for integer values of $x_1, \cdots, x_n$ take integer values. Then the transformation

$$y_i \to y_i - f_i(x_1, \cdots, x_n) \qquad (i = 1, \cdots, n) \tag{1.3}$$

takes (1.1) into

$$\sum_{i=1}^{n} x_if_i(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_iy_i, \tag{1.4}$$

and the complete integer solution of (1.4) is

$$\begin{aligned} x_i &= \alpha\alpha_i, \\ y_i &= -\sum_{j=1}^{i-1} \alpha_i\beta_{j,i} + \sum_{j=1}^{n-i} \alpha_{i+j}\beta_{i,i+j} + f_i(\alpha\alpha_1, \cdots, \alpha\alpha_n). \end{aligned} \tag{1.5}$$

These solutions are valid in any Euclidean ring, as may be seen from the proof (see footnote 1) of (1.2). The like, therefore, holds for equations and their solutions obtained from (1.1), (1.4) by operating within any given Euclidean ring.

Equations of the types (1.6)–(1.11) are to be considered.

$$a_1x_1 \cdots x_{i_1} + a_2y_1 \cdots y_{i_2} + \cdots + a_nz_1 \cdots z_{i_n} = 0, \tag{1.6}$$

in which $a_1, \cdots, a_n$ are constant integers $\neq 0$ and $i_1 > 1, i_2 > 1, \cdots, i_n > 1$. This is one possible generalization of (1.1); its complete integer solution proceeds from (1.2).

$$\sum_{j=1}^{n} a_jx_{j1} \cdots x_{ji_j}f_j(x_{j1}, \cdots, x_{ji_j}) = \sum_{j=1}^{n} a_jx_{j1} \cdots x_{ji_j}y_j, \tag{1.7}$$

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[1] Th. Skolem, *Diophantische Gleichungen*, 1938, p. 20. The form of this solution is considerably simpler than that given by the method of L. Aubry, *Réponse á la solution générale par identités de l'équation par V. G. Tariste*, L'Intermédiare des Mathématiciens, vol. 23 (1916), pp. 133–134, reproduced in Dickson (see footnote 2), p. 194.