AN EXPLICIT FORMULA FOR THE SOLUTION OF THE ULTRAHYPERBOLIC EQUATION IN FOUR INDEPENDENT VARIABLES

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1. **Introduction.** Equations of the form

$$\sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} - \sum_{i=1}^{n} \frac{\partial^2 u}{\partial y_i^2} = 0 \qquad (n \geqq 2, m \geqq 2) \tag{1.1}$$

are referred to as "ultrahyperbolic partial differential equations". In 1901, G. Hamel [2][1] investigating the problem of finding all geometries in which the straight lines are the shortest ones considered an equation equivalent to (1.1) $(n = m = 2)$; he established the existence of a function which satisfied the equation in the neighborhood of the intersection of two characteristic hyperplanes and which assumed prescribed initial values on these planes, but the initial values are restricted to be analytic in two of their three arguments. It has only been in recent years that properties of the solutions of (1.1) have been discovered that are not restricted by analyticity.

In 1932, L. Ásgeirsson [1] discovered a mean-value theorem which applies to any twice continuously differentiable solution of (1.1) $(n = m)$. By the use of this mean-value theorem, it has been shown that on a non-characteristic hyperplane the values for any solution of (1.1) cannot be arbitrarily assigned so as to furnish a solution.[2]

In 1938, F. John [3], using Ásgeirsson's theorem, was able to determine the most general solution, u, of (1.1) $(n = m = 2)$ existing in all space; interpreting the independent variables as suitable functions of the Plücker coordinates of a line in 3-dimensional space, u becomes a function of lines and Ásgeirsson's theorem takes the following form. If the line function u is a solution of (1.1) $(n = m = 2)$, then for every hyperboloid H of revolution and of one sheet, the mean values of u for the two families of generating lines of H are equal. Calling a function of the lines of 3-dimensional space with the above property harmonic, John demonstrates that every harmonic line function which is twice continuously differentiable is equivalent to a solution of (1.1) $(n = m = 2)$. It is also shown that the line integrals of a sufficiently regular point function form a harmonic line function and that a sufficiently regular harmonic line function is representable as line integrals of a point function.

The present paper considers the equation (1.1) $(n = m = 2)$. The variables of this equation are regarded as the coordinates of a 4-dimensional point that varies in a domain defined by an initial hypersurface and the characteristic

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[1] The numbers in brackets refer to the bibliography.

[2] Ásgeirsson's results and the applications of his mean-value theorem are to be found in the book *Methoden der Mathematischen Physik*, vol. II, by R. Courant and D. Hilbert.