FINITE GROUPS AND RESTRICTED LIE ALGEBRAS

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1. **Introduction.** We are concerned with a method of Zassenhaus[1] which associates with abstract algebraic groups certain Lie rings.

By a Lie ring L over a field K, we mean a linear space (of finite or infinite dimension) over K, in which there is defined an operation $[x, y]$, called the commutator of the two elements x and y, satisfying

(a) $[x, y]$ is in L when x and y are in L.

(b) $[mx + ny, z] = m[x, z] + n[y, z]$, m, n in K, and a similar expression with the sum on the other side of the comma.

(c) $[x, x] = 0$ (and so $[x, y] = -[y, x]$).

(d) $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$.

If L has a finite basis over K, we indicate this fact by calling L a Lie *algebra* over K.

If K is of characteristic p, we say that L is of characteristic p. An important class of Lie rings of characteristic p is the "restricted" Lie rings.[2] L is a restricted Lie ring if it has characteristic p, and if there is associated with every element x of L an element denoted by x^p which satisfies the condition

(e) $[y, x^p] = [\cdots \overbrace{[[y, x], x], \cdots, x]}^{p}$ for all y in L.

Let L and L' be two Lie rings over K with a one-to-one mapping $x \to x'$ of L onto L', such that $mx + ny \to mx' + ny'$ (m, n in K) and $[x, y] \to [x', y']$. Then L and L' are said to be isomorphic. If L and L' are restricted Lie rings and, in addition, $x^p \to (x')^p$, we shall say that L and L' are "p-isomorphic".

Let L be any Lie ring of characteristic p. We may or may not be able to choose for each element x an element x^p satisfying condition (e). If we are able to do so, we follow the nomenclature of Zassenhaus and call L a p-invariant Lie ring. For each element x there may exist several elements which would satisfy the condition (e). Indeed, let x' be such an element and C be the centrum (set of elements c such that $[c, y] = 0$ for all y in L); then $x' + c$ also satisfies (e) for any c in C.

Given a p-invariant Lie ring L, by choosing, for each element x, one element x^p which satisfies condition (e), L becomes a restricted Lie ring. Clearly we can begin with the same L and, by choosing different elements x^p, get restricted Lie rings which are not p-isomorphic. A methodical way of choosing the x^p is

Received April 24, 1941.

[1] H. Zassenhaus, *Ein Verfahren, jeder endlichen p-Gruppe einer Lie-Ring mit der Charakteristik p zuzuordnen*, Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität, vol. 13(1939), pp. 200–207.

[2] N. Jacobson, *Abstract derivation and Lie algebras*, Transactions of the American Mathematical Society, vol. 42(1937), pp. 206–224.