POSITIVE SOLUTIONS OF BINOMIAL INEQUALITIES

By Joseph Miller Thomas

1. **Introduction.** Let S be a finite set of inequalities of the form

$$m > p, \tag{1.1}$$

where m, p are monomials in the unknowns $x_1, x_2, \cdots, x_n$ with positive coefficients. We give a method of testing S for consistency and solving it in the set of all positive numbers. Although the sign $>$ is employed throughout the discussion of §§1–5, the method is valid for an arbitrary distribution of $>$, $\geqq$, $=$, except that the restriction to $>$ is essential in Theorem 4.1, as it is in §6. When various signs occur, they must be multiplied in accordance with the following table.

	$>$	$\geqq$	$=$
$>$	$>$	$>$	$>$
$\geqq$	$>$	$\geqq$	$\geqq$
$=$	$>$	$\geqq$	$=$

In other words, although $\geqq$ and $>$ when multiplied imply both $\geqq$ and $>$, the stronger inequality is defined as the product.

The order of the terms in an equality $m = p$ is not determined. When $m = p$ is present, both $m = p$ and $p = m$ must be included, if elimination is to proceed by multiplication alone. The method is, of course, valid for systems composed entirely of equations.

The method is also applicable when the exponents, instead of being non-negative integers, are any real numbers. As a consequence, solving any finite set of linear inequalities in the real field is equivalent to solving a system (1.1) in the positive numbers. From this standpoint the method can be identified with the process of elimination developed by Dines.[1]

2. **Reduction processes.** If S contains in addition to (1.1) the inequality

$$q > t, \tag{2.1}$$

then S implies

$$(m - p)(q - t) > 0,$$

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[1] L. L. Dines, *Systems of linear inequalities*, Annals of Mathematics, (2), vol. 20(1918–1919), pp. 191–199.