CONJUGATE NETS AND ASSOCIATED QUADRICS

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1. **Introduction.** This paper is concerned with the study of the projective differential geometry of certain quadrics which are associated with a point of a curve of a conjugate net on an analytic surface in ordinary space. Some portions of the theory of a surface referred to a conjugate net N_x are summarized in §2, where the differential equation of a general conjugate net N_λ, distinct from the parametric conjugate net N_x, is written. The two curves of the net N_λ that pass through a point of the surface will be denoted by C_λ and C_μ. In §3, power series expansions in non-homogeneous projective coördinates for the curve C_λ are computed to terms of as high degree as will be needed in this paper. Some immediate geometrical applications to these series are made. For example, in §4, certain pencils of quadrics having contact of the second order at a point of the surface are investigated. Then, in the following section, the quadrics of Moutard at a point of the surface and in the directions of the tangents to the curves of the net N_λ through the point are considered. Necessary and sufficient conditions for a curve C_λ to be a plane curve and for C_λ to be a cone curve are given in §6. In the next section two quadrics which are associated with each point of a curve C_λ are defined and the equations of these quadrics are found.

2. **Analytic basis.** Let the projective homogeneous coördinates $x^{(1)}, \cdots, x^{(4)}$ of a point P_x on a surface S referred to a conjugate net N_x in ordinary space be given as analytic functions of two independent variables u, v by equations of the form

$$x = x(u, v). \tag{1}$$

Then the coördinates x of a point on the surface and the coördinates y of the point which is the harmonic conjugate of the point x with respect to the foci of the axis of the point x satisfy a system of equations of the form[1]

$$\begin{aligned} x_{uu} &= px + \alpha x_u + Ly, \\ x_{uv} &= cx + ax_u + bx_v, \\ x_{vv} &= qx + \delta x_v + Ny \qquad (LN \neq 0). \end{aligned} \tag{2}$$

It is easily verified that

$$y_u = fx - nx_u + sx_v + Ay, \qquad y_v = gx + tx_u + nx_v + By, \tag{3}$$

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[1] E. P. Lane, *Projective Differential Geometry of Curves and Surfaces*, Chicago, 1932, p. 138.