INVARIANTS OF A SYSTEM OF LINEAR HOMOGENEOUS DIFFERENTIAL EQUATIONS OF THE SECOND ORDER

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Introduction. E. J. Wilczynski, in his treatise *Projective Differential Geometry of Curves and Ruled Surfaces* (Leipzig, Teubner, 1906), Chapter IV, discusses the invariants of a system of linear homogeneous differential equations under a projective transformation. He finds first the induced transformation of the coefficients of a system of n linear homogeneous differential equations (in n dependent variables) of the m-th order and then proceeds to calculate the invariants for the particular case of $m = 2$, $n = 2$. This calculation, as presented there, requires the solution of a complete system, a process which is quite laborious even for the special case considered by Wilczynski. This method becomes more difficult if we try to calculate the invariants in the case $m = 2$, $n = 3$; and the calculations become more and more involved as the number of variables becomes larger.

It is our purpose to present an easier and more elegant solution of the problem of finding the invariants of a system of n linear homogeneous differential equations of the second order under a linear transformation. Consider the system

$$y_i'' + \sum_{j=1}^{n} L_{ij}(x)y_j' + \sum_{j=1}^{n} M_{ij}(x)y_j = 0 \qquad (i = 1, 2, \cdots, n),$$

where $y_i' = dy_i/dx$, $y_i'' = d^2y_i/dx^2$, under the transformation

$$y_i = \eta_i + \sum_{j=1}^{n} K_{ij}(x)\eta_j \qquad (i = 1, 2, \cdots, n).$$

We find in this paper the invariants which are functions of the arguments L_{ij}, $L'_{ij} = dL_{ij}/dx$, M_{ij} and also the invariants which are functions of the arguments L_{ij}, L'_{ij}, L''_{ij}, M_{ij} and M'_{ij}.

Just as Wilczynski applied the theory for the case $n = 2$ to the study of ruled surfaces, we could also apply the results given in this paper to the study of a certain type of projective configuration represented by the system of differential equations here considered. However, this will be reserved for another work.

1. **Notation.** For the sake of simplicity we shall find it convenient to introduce the following permanent notations.

The small letters, for example y, η, shall represent vectors, that is, they will stand respectively for y_i, η_i $(i = 1, 2, \cdots, n)$. The letters x and a shall

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