THEORY OF COGROUPS

By J. E. Eaton

1. **Introduction.** Grouplike systems with non-unique multiplication have been the subject of several recent papers. In 1938 Dresher and Ore[1] undertook an axiomatic investigation of such systems, which they called multigroups. Some of their most interesting results were concerned with the relation of submultigroups of a multigroup to the multigroup itself. However, for these theorems they found it necessary to restrict their consideration to submultigroups which satisfied a "reversibility" condition.

In this paper we shall examine some of the properties of a special type of multigroup in which every submultigroup is reversible. We have called this particular kind of multigroup a *cogroup* (or, more properly, *left cogroup*). Since the multiplicative system of the left coset decomposition[2] of any group with respect to a subgroup is a cogroup, a few of the results contained in this paper may be of some interest from a group theoretical viewpoint. However, if it can be shown that any cogroup may be generated by the left coset decomposition of a group, many of our theorems would reduce to trivialities. Such a proof, nevertheless, would be of considerable importance. It would permit a formulation of the problem of the extension of groups by non-normal subgroups analogous to the so-called solution of Schreier's for the normal case.

2. **Axioms.** A *cogroup* (or, more properly, *left cogroup*) is an algebraic system in which there is defined a single binary operation, multiplication, subject to six axioms.

Axiom 1. The Product. *If c_i and c_j are any two elements of a cogroup $\mathfrak{C}$, then the product c_ic_j is a non-void subset of $\mathfrak{C}$.*

$$c_ic_j = \{c_k'\}.$$

The existence of the product of any two elements of $\mathfrak{C}$ permits us to give meaning to the notion of the product of any two subsets of $\mathfrak{C}$. If A and B are two non-void subsets of $\mathfrak{C}$ with elements $\{a_i\}$ and $\{b_i\}$ respectively, then an element c of $\mathfrak{C}$ is in AB if and only if c is contained in some product a_jb_k.

Axiom 2. The Associative Law. *If c_i, c_j, c_k are any three elements of $\mathfrak{C}$, then*

$$(c_ic_j)c_k = c_i(c_jc_k).$$

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[1] Dresher and Ore, *Theory of multigroups*, American Journal of Mathematics, vol. 60(1938), pp. 705–733.

[2] We shall call a *left coset* of a subgroup $\mathfrak{H}$ a complex of the form $\mathfrak{H}g$.