GENERALIZATIONS OF NON-ALTERNATING AND NON-SEPARATING TRANSFORMATIONS

By E. P. Vance

1. **Introduction.** In two papers, one by G. T. Whyburn[1] and the other by J. F. Wardwell,[2] non-alternating and non-separating transformations have been discussed. Two other transformations of the same type, completely non-alternating and completely non-separating, have also been considered but as yet have not appeared in the literature. The conditions on these four transformations are quite strong, thus limiting the applications. For example, no transformation on an acyclic space can possibly satisfy the hypothesis of completely non-separating. Also, if the transformation is non-separating, then the image is always cyclically connected. Hence it seems desirable to weaken the conditions on these four transformations, thus extending the range of possible applications. It is the purpose of this paper to present what seems to be a natural weakening or generalization of these four transformations, to investigate what theorems concerning the original stronger transformations carry over, and in most cases to consider what additional hypothesis must be added to the weakened transformations to obtain similar results.

Throughout this paper it will be assumed that the space A is a compact metric continuum. All transformations $T(A) = B$ which are considered are assumed to be single valued and continuous.

Six new transformations are defined in this paper. A transformation T is said to be *weakly non-alternating*[3] if, for any two distinct points x and y of B, $T^{-1}(x)$ does not separate two points of $T^{-1}(y)$ non-degenerately in A.[4] A transformation T is said to be *weakly non-separating*[5] if, for any point x of B, $T^{-1}(x)$ does not separate any two points of A non-degenerately. A transformation T is said to be *weakly completely non-alternating*[6] if, for any two points x and y

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[1] G. T. Whyburn, *Non-alternating transformations*, American Journal of Mathematics, vol. 56(1934), pp. 294–302.

[2] J. F. Wardwell, *Non-separating transformations*, this Journal, vol. 2(1936), pp. 745–750.

[3] A transformation T is said to be *non-alternating* if, for any two distinct points x and y of B, $T^{-1}(x)$ does not separate $T^{-1}(y)$ in A. G. T. Whyburn, loc. cit., p. 294.

[4] A subset K will be said to *separate* a subset H *non-degenerately* if K separates H in A and no single point of K separates H in A.

[5] A transformation T is said to be *non-separating* if, for any point x of B, $T^{-1}(x)$ does not separate A. J. F. Wardwell, loc. cit., p. 745.

[6] A transformation T is said to be *completely non-alternating* if, for any two points x and y of B and any closed subset K of $T^{-1}(x)$, K does not separate $T^{-1}(y)$ in A.