A CLASS OF CONTINUED FRACTIONS

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Introduction. The algebraic continued fractions

$$(1) \qquad \frac{k_1 z|}{|1} + \frac{k_2 z|}{|1} + \cdots, \qquad k_i \text{ complex and } \neq 0,$$

whose coefficients satisfy the condition

$$(2) \qquad \sum_{i=1}^{\infty} |k_i| < \infty$$

possess the following interesting property:

The numerators and denominators of the approximants of (1) *form respectively two sequences that converge uniformly over any bounded region of the z-plane* [3].[1]

A more general class than (1) is given by

$$(3) \qquad \frac{k_1 \quad |}{|z - c_1} + \frac{k_2 \quad |}{|z - c_2} + \cdots, \qquad k_i \text{ and } c_i \text{ complex}, \qquad k_i \neq 0.$$

In fact, except for simple changes of the variable and the fraction, (1) can be obtained from (3) by taking all $c_i = 0$ ([4], §61). In this paper we study some consequences of condition (2) for this general class of continued fractions.

The general case of unrestricted c_i is considered briefly in §1. A generalization of the above-mentioned property of (1) under (2) in this case is given in

Theorem 1. *Denote the n-th approximant of* (3) *by* $P_n(z)/Q_n(z)$. *If we have* (2), *then the two sequences*

$$(4) \qquad \frac{P_n(z)}{\prod_{i=1}^{n} (z - c_i)}, \qquad \frac{Q_n(z)}{\prod_{i=1}^{n} (z - c_i)} \qquad (n = 1, 2, \cdots)$$

converge each uniformly in every domain[2] *of the z-plane at a positive distance from* $\{c_i\}$*—the set of points* c_i $(i = 1, 2, \cdots)$.

More precise results are obtained in §§2 and 3 for a number of special cases. Convergence properties of (3) are discussed in §4. In §5 we consider the special case corresponding to the condition

$$(5) \qquad \sum_{i=1}^{\infty} \frac{1}{|c_i|} < \infty.$$

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[1] Numbers in brackets refer to the bibliography.

[2] By *domain* we mean a point set in the extended complex plane.