CONTRIBUTIONS TO THE THEORY OF GROUPS OF FINITE ORDER

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The present paper contains a number of results of diverse nature in the theory of finite groups. One may say that the guiding principle is the application of structure theory to the theory of groups. In two recent papers[1] I have already shown that this method is very useful for various investigations in group theory. In the present paper this point of view is particularly important in the study of *non-normal* chains of subgroups of groups, a field which seems to be untouched until now. One of the main problems which is solved by this structural approach is the determination of all groups in which *every* chain of subgroups, with each term maximal, but usually not normal, in the preceding shall have the Jordan-Hölder property that the indices are the same in some order.

Let us also make the following general remark. Groups are ordinarily defined by their elements, or, equivalently in structural terms, each subgroup is the union of cyclic groups, and hence the group properties are naturally stated by means of element properties. By dualizing this process one is led to the investigation of the properties of a group in relation to its maximal subgroups and several of the results of this paper may be said to belong to this category.

In the first chapter various properties of permutable groups are derived and particularly the existence of permutable decompositions is investigated. It is shown that if a group has the property that all maximal subgroups are permutable, they are all normal and the group is nilpotent. The concept of quasinormality introduced in the preceding papers is studied further, and it is shown that in several cases normality and quasi-normality are identical concepts. In particular, no simple groups can contain quasi-normal subgroups.

In the second chapter properties of normal decompositions as union and the dual decomposition as cross-cut are considered and the relation to the representations of the group as a permutation group is pointed out. Here one also finds the determination of the ϕ-group of a group.

In the third chapter properties of arbitrary non-normal chains of subgroups are deduced. It is shown that to any complete chain there exists a chain with the same index type passing through any prescribed principal chain. This result reduces the study of various properties of such chains to the case of simple groups.