NOTE ON TOPOLOGICAL MAPPINGS

By J. H. Roberts

E. W. Miller[1] has given an example of an acyclic curve M such that if f is any topological mapping of M into a subset of itself, then $f(M) = M$. R. Baer[2] has given an example of an acyclic curve M such that if f is a topological function and $f(M) = M$, then f is the identity. Neither of these examples has *both* of the properties mentioned above. O. Hamilton[3] has raised the question as to whether or not any acyclic curve has both the above properties. The present paper answers this question in the affirmative by describing a compact acyclic continuous curve H such that the only topological function mapping H into a subset of itself is the identity.

Now Menger's[4] "universal tree of order 4" is made up as follows: (1) There is a single interval S which is called the interval of the "0-th degree". (2) For each point P of a countable set T_0 dense on S, but not containing an end-point of S, there are two intervals having P as end-point, these intervals being of the 1-st degree. (3) In general, for every $n \geqq 0$ there is a countable set T_n dense on every interval of the n-th degree and for each point P of T_n there are two intervals having P as end-point, these intervals being the intervals of the $(n + 1)$-th degree. (4) The curve M is the sum of all the intervals of all the different degrees, plus all limit points of this sum.

Our curve H will be defined as a subset of Menger's curve M. To get H we modify M in this way: Having decided that a certain interval I of degree r (in M) is to be in H, we may wish to have only *one* interval of degree $r + 1$ for each of the junction points on I. In this case we select arbitrarily (to be a part of H) one of the two intervals of degree $r + 1$ ending in each junction point on I. In the future we will indicate this by writing "the junction points on I are to be of order 3 in H".

It is convenient to use the following notation: Suppose P is a junction point of M on an arc of degree r. Then an arc I of degree $> r$ is said to "join on through P" if P separates I (or $I - P$) from S (or $S - P$).

We now set up a 1-1 correspondence between the set of all finite permutations of positive integers and the integers of the form 2^k. Let $x_{ij\cdots k}$ be the

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[1] *The Zarankiewicz problem*, Bull. Amer. Math. Soc., vol. 38(1932), pp. 831–834.

[2] *Beziehungen zwischen den Grundbegriffen der Topologie*, Sitzungsberichte der Heidelberger Akademie der Wissenschaften, 1929, no. 15.

[3] *Fixed points under transformations of continua*, Trans. Amer. Math. Soc., vol. 44(1938), pp. 18–24; especially p. 24.

[4] *Kurventheorie*, Leipzig, 1932, p. 318.