THE MEASURE OF GEODESIC TYPES ON SURFACES OF NEGATIVE CURVATURE

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1. **Introduction.** Various problems arise in connection with the transitivity of a dynamical system. The first of these concerns the existence of transitive motions. Secondly, if there exist such motions, what is the measure of the totality of transitive motions? Thirdly, is the system metrically transitive? If the last holds, almost all the motions are transitive, so that each of the first two of the above properties is a consequence of the following one.

The first of these problems has been solved in the case of the geodesic problem on a class of surfaces of negative curvature and even for surfaces with some positive or zero curvature, provided there is not enough to destroy the instability of the geodesics (cf. Morse [1], Hedlund [1]). The second and third problems have been solved only for a restricted subclass of these surfaces, namely, those of constant negative curvature, of finite area, and of finite connectivity (cf. E. Hopf [1]). The constancy of the curvature plays an important rôle in the proofs of these results, for it implies that certain transformations are analytic and thus transform sets of measure zero into sets of measure zero. It is the lack of information concerning the corresponding transformations in the variable case which causes the difficulty in applying the methods of the constant case.

This paper gives a solution of the second problem for a class of surfaces of negative curvature, not necessarily constant, of finite area and finite connectivity. This class includes, in particular, all closed orientable surfaces of negative curvature. It also includes surfaces with "parabolic" openings. It is shown that on all surfaces of the stated class almost all the geodesics are transitive. The extension of this result to non-orientable surfaces is easily proved.

In the definition of the preceding class of surfaces, a Fuchsian group of the first kind is used. If the defining Fuchsian group is of the second kind, the geodesics behave in an entirely different manner. It is shown that in this case almost all the geodesics are unstable in the sense that for both future and past time they eventually remain outside any fixed finite region of the surface. (Cf. E. Hopf [1] for results of this kind in the case of constant negative curvature.) This class includes surfaces with "hyperbolic" openings similar to the surfaces which Hadamard (cf. Hadamard [1]) constructed. The class does not include all these Hadamard surfaces, however, so that it is not possible to say that in all cases the perfect sets of geodesics discovered by Hadamard are sets of measure zero. This problem will be taken up in a later paper.

The method used is similar to that used by Tuller (cf. Tuller [1]) in attaining

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