HEREDITARY ARC SUMS

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1. The object of this paper is to obtain conditions under which a compact locally connected continuum and all of its subcontinua are arc sums. A continuous curve possessing this property will be called a *hereditary arc sum*. G. T. Whyburn[1] has shown that a necessary and sufficient condition that a curve be an arc sum, that is, the sum of a countable number of simple continuous arcs, is that it have a countable number of end-points and that each true cyclic element be an arc sum. It is well known that if an acyclic curve is an arc sum, every curve contained in it is an arc sum. The corresponding proposition about continuous curves in general, or even for the class of cyclic curves, is not true, as is easily shown by examples.

It is proved in this paper that the property of being a hereditary arc sum is equivalent to each of the following:

(1) *each acyclic curve in the given curve is an arc sum*;

(2) *the boundary of each region in the curve is a countable point set*;

(3) *every connected subset is a* G_δ;

(4) *the non-local separating points are countable*;

(5) *for every connected subset* G, $\bar{G} - G$ *is countable.*

It is shown that every curve containing a convergence continuum contains an acyclic non-arc-sum, hence that the class of hereditary arc sums is contained in the class of hereditarily locally connected continua. In §5 it is proved that the property of being a hereditary arc sum is invariant under continuous transformations which are either monotone or interior.

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2. It is evident that each acyclic curve in M is an arc sum if M is a hereditary arc sum. To establish the converse we prove first

Theorem A. *Every hereditarily locally connected compact continuum which is not an arc sum contains an acyclic non-arc-sum.*

Proof. It will be convenient to introduce the notion of an *adjacent cycle point*. A point p is called an adjacent cycle point of the set M provided that every neighborhood of p contains a simple closed curve lying in M. Clearly, the set of adjacent cycle points is closed. Let M_1 be the set of adjacent cycle

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[1] G. T. Whyburn, *Concerning continuous curves and arc sums*, Fundamenta Mathematicae, vol. 14(1929), pp. 103–106.