THE SUBGROUP OF ORDER n OF A TRANSITIVE GROUP OF DEGREE n AND CLASS $n - 1$

By Louis Weisner

It is known that a transitive permutation group of degree n and class $n - 1$ has an invariant subgroup of order n consisting of the identity and $n - 1$ permutations of degree n.[1] The object of this paper is to demonstrate that this subgroup of order n is an *Abelian* group.[2]

A permutation of degree $n - 1$ of a transitive permutation group of degree n and class $n - 1$ generates, with the unique subgroup N of order n, a group G which is also a transitive permutation group of degree n and class $n - 1$. We shall confine our attention to this group G. If the order of G is mn, a subgroup M of G that leaves one symbol fixed is a cyclic group of order m,[3] and $G = \{M, N\}$. Let G' be an abstract group simply isomorphic with G, and let M' and N' be the subgroups of G' that correspond to M and N, respectively.

Since N is the commutator subgroup of G, G' has exactly m distinct representations in one variable. Denote these by $\Gamma_1, \cdots, \Gamma_m$, where Γ_1, as usual, denotes the identical representation. Let $\Gamma_{m+1}, \cdots, \Gamma_r$ be the other distinct irreducible representations of G', and let n_v be the number of variables operated on by Γ_v.

The relation among group characters

$$\sum_{v=1}^{r} \chi_i^{(v)} \chi_{i'}^{(v)} = g/h_i,$$

where h_i is the number of elements in the i-th conjugate set of G and g is the order of G, becomes, for an element $A \neq 1$ of M',

$$\sum_{v=1}^{r} \chi^{(v)}(A)\chi^{(v)}(A^{-1}) = m.$$

The terms which arise from $v = 1, \cdots, m$ have the value 1; hence

$$\chi^{(v)}(A) = 0 \qquad (v = m + 1, \cdots, r). \tag{1}$$

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[1] G. Frobenius, *Ueber auflösbare Gruppen* IV, Sitzungsberichte Berlin, 1901, pp. 1223–1225; A. Speiser, *Theorie der Gruppen von endlicher Ordnung*, 3d edition, 1937, p. 202.

[2] The theorem has been proved for the case in which the subgroup that leaves one symbol fixed is of even order. See W. Burnside, *Theory of Groups of Finite Order*, 2d edition, 1911, p. 172.

[3] We could take m to be a prime number, but no advantage is to be derived therefrom.