ALGEBRAIC FUNCTIONS OF ANALYTIC ALMOST PERIODIC FUNCTIONS

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It is our purpose here to extend to analytic almost periodic (a. p.) functions investigations previously carried on by several authors concerning the a. p. character of the solutions of algebraic equations whose coefficients are a. p. functions of a real variable.

The basic theorem (Theorem 1) concerning the existence of analytic a. p. solutions is an almost immediate consequence of the corresponding theorem (due to Walther and Cameron) for the real case, which latter may be stated as follows: If the coefficients of the equation

$$y^m + X_1(t)y^{m-1} + \cdots + X_m(t) = 0$$

are a. p. functions of the real variable t, and if the absolute value of the discriminant is bounded from zero, then the equation has m distinct a. p. solutions. Theorem 1 is proved in §1.

Further information concerning the nature of the solutions can be obtained by taking account of the exponents of the exponential series of the coefficients. Thus in the real case Cameron showed that the modulus of the Fourier exponents of each solution is contained in the quotient by an integer ($\leqq m$) of the modulus of the Fourier exponents of the coefficients. In a previous paper[1] we have refined this result by means of the notion of the "almost translation group" of the equation; particularly in the case where this group is transitive. As these results can be carried over directly to the analytic case, we shall not enter on them further, but refer the reader to the paper cited.

In the analytic case particular importance attaches to functions whose Dirichlet exponents are bounded on one side, since the functions are then a. p. in a half-plane. In §2 we show that when the exponents of the coefficient functions are bounded below and those of the discriminant have a minimum, then the exponents of the solutions are bounded below (Theorem 2). If in addition the exponents of the last coefficient likewise have a minimum, then the exponents of every solution also have a minimum.

Ostrowski[2] has treated the problem from quite another point of view, namely, the purely formal one, where the coefficients are taken to be formal Dirichlet

Received July 10, 1938.

[1] H. Bohr and D. A. Flanders, *Algebraic equations with almost-periodic coefficients*, Kgl. Danske Videnskabernes Selskab, Mathematisk-fysiske Meddelelser, vol. 15(1937), pp. 1–49.

[2] A. Ostrowski, *Über algebraische Funktionen von Dirichletschen Reihen*, Mathematische Zeitschrift, vol. 37(1933), pp. 98–133.