SOLUTIONS OF A DIFFERENTIAL EQUATION OF THE FIRST ORDER AND FIRST DEGREE IN THE VICINITY OF BRANCH POINTS OF THE SOLUTION

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Introduction. In this paper a method previously developed[1] for solving systems of differential equations about an ordinary point is applied to the equation

$$(1) \qquad 2x\frac{dx}{dt} = \sum_{h=0}^{\infty} f_h(t)x^h \qquad (f_0(t) \not\equiv 0),$$

which has a singularity at $x = 0$.

The coefficients $f_h(t)$, which for convenience will be represented by f_h, are assumed to satisfy the following conditions:

I. The functions f_h $(h = 1, 2, \cdots)$ are real and are dominated on the interval (t_0, t_1) by a function f of t. The functions f_h $(h = 0, 1, \cdots)$ are integrable (Riemann) and their only points of discontinuity belong to a set E of measure zero.

II. The function f is integrable (Riemann) on the interval (t_0, t_1) and is equal to unity for all values of $t < t_0$. The points of discontinuity of f belong to the set E. The definite integral of f on the interval (t_0', t_0) where $t_0' \leqq t_0$ is represented by

$$(2) \qquad \int_{t_0'}^{t_0} f\,dt = t_0 - t_0' = c \geqq 0.$$

III. The function f_0 is real and satisfies the inequalities

$$(3) \qquad \begin{cases} 0 \leqq \displaystyle\int_{t_0}^{t} f_0\,dt \leqq \int_{t_0}^{t} f\,dt, \\[2ex] \dfrac{1}{\displaystyle\int_{t_0}^{t} f_0\,dt + c} \leqq \dfrac{A^2}{\displaystyle\int_{t_0}^{t} f\,dt + c} \equiv \dfrac{A^2}{F^2}, \end{cases}$$

where A is a real constant satisfying the inequality $A \geqq 1$ and the zeros of the function F belong to the set E.

1. **Formal solutions of the differential equation** (1). The transformation

$$(4) \qquad x = \sum_{h=1}^{\infty} y_h K^h,$$

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[1] J. Pierce, *Solutions of systems of differential equations in terms of infinite series of definite integrals*, this Journal, vol. 3(1937), pp. 616–622.