APPROXIMATION TO THE SOLUTION OF A NORMAL SYSTEM OF ORDINARY LINEAR DIFFERENTIAL EQUATIONS

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1. **Introduction.** This note is concerned with certain problems of approximation on a given finite interval $a \leqq t \leqq b$ to the solution of the system of ordinary equations

$$(1) \qquad \begin{cases} \dfrac{dx_i}{dt} = \theta_{i1}(t)x_1 + \cdots + \theta_{im}(t)x_m + \theta_i(t), \\ x_i(t_0) = c_i, \end{cases} \qquad (i = 1, \cdots, m),$$

where $a \leqq t_0 \leqq b$. Let there be given an infinite sequence of functions $\varphi_i(t)$ which are defined and linearly independent (in finite subsets) on (a, b). Let

$$y_{in_i}(t) = c_{i1}\varphi_1(t) + \cdots + c_{in_i}\varphi_{n_i}(t) \qquad (i = 1, \cdots, m).$$

One may consider the problem of approximating to the solution of the system (1) by means of a set of m linear combinations $y_{in_i}(t)$ satisfying the initial conditions so as to minimize the sum

$$(2) \qquad \int_a^b |y'_{1n_1} - \theta_{11}y_{1n_1} - \cdots - \theta_{1m}y_{mn_m} - \theta_1|^{r_1}\,dt + \cdots + \int_a^b |y'_{mn_m} - \theta_{m1}y_{1n_1} - \cdots - \theta_{mm}y_{mn_m} - \theta_m|^{r_m}\,dt,$$

where the r_i are given constants > 0. Another problem is that of approximating to the solution by means of a set of m linear combinations $y_{in_i}(t)$ so as to minimize

$$(3) \qquad \sum_{i=1}^{m} a_i\,|x_i(t_0) - y_{in_i}(t_0)|^{r_{m+i}} + \int_a^b |y'_{1n_1} - \theta_{11}y_{1n_1} - \cdots - \theta_{1m}y_{mn_m} - \theta_1|^{r_1}\,dt + \cdots + \int_a^b |y'_{mn_m} - \theta_{m1}y_{1n_1} - \cdots - \theta_{mm}y_{mn_m} - \theta_m|^{r_m}\,dt,$$

where the r_i and the a_i are given constants > 0. Under further suitable hypotheses regarding the functions involved in these problems, questions of existence and uniqueness of approximating sets of functions will be discussed. The problem of uniform convergence as $n_i \to \infty$ of the $y_{in_i}(t)$ to the $x_i(t)$ will be discussed only in case the y_{in_i} are polynomials of degree at most n_i in t.

A number of papers[1] have been written recently dealing with similar problems

Received October 7, 1937.

[1] W. H. McEwen, Trans. Amer. Math. Soc., vol. 33(1931), pp. 979–997; Bulletin Amer. Math. Soc., vol. 38(1932), pp. 887–894. For other references to the literature see these papers by McEwen.