FUNCTIONS OF INTEGRABLE SQUARE IN SEVERAL COMPLEX VARIABLES

By S. Bochner

As in two previous notes[1] we consider in the space C_k of k complex variables

$$z = (z_1, \cdots, z_k), \qquad z_\kappa = x_\kappa + iy_\kappa,$$

point sets of a special nature which we call *tubes*. A point set T of C_k is a tube, if there exists a point set S in the space R_k of real variables $x = (x_1, \cdots, x_k)$ such that T consists of all k-dimensional planes

$$(1) \qquad x_\kappa = x_\kappa^0 \qquad (-\infty < y_\kappa < \infty; \kappa = 1, \cdots, k)$$

for which $(x_1^0, \cdots, x_k^0)$ is any point of S. The set S is called the basis of T, and we also denote T more explicitly by T_S. The tube T_S is open or closed in C_k if and only if S is open or closed in R_k; it is convex if and only if S is convex, and the convex hull[2] $\tilde{T}$ of a tube T is again a tube whose basis $\tilde{S}$ is the convex hull of S.

We say that a function $f(z) = f(z_1, \cdots, z_k)$ is of integrable square in T if the function

$$f_x(y) = f(x_1 + iy_1, \cdots, x_k + iy_k)$$

belongs to the Lebesgue class L_2 over the y-space, for every $x \subset S$, and if moreover there exists a constant K such that

$$(2) \qquad \int_{-\infty}^{\infty} \cdots \int |f_x(y)|^2 dv_y \leqq K,$$

for all $x \subset S$, the symbol dv_y denoting the Euclidean volume element $dy_1 \cdots dy_k$.

In our first note we proved the following theorem. If $f(z)$ is analytic and of integrable square in an open tube T, then it also exists and is of integrable square in $\tilde{T}$. In the present paper we shall extend this theorem to the case of tubes which are not necessarily open.

Assumptions. (1) *The basis S is such that any two points P, Q of S have a finite Euclidean distance $D(P, Q)$ on S in the following sense. Corresponding to*

Received May 6, 1938.

[1] S. Bochner, *Bounded analytic functions in several variables and multiple Laplace integrals*, American Journal of Math., vol. 59(1937), pp. 731–738; *A theorem on analytic continuation of functions in several variables*, Annals of Math., vol. 39(1938), pp. 14–19. I am indebted to H. Behnke for pointing out to me that the theorem of the second note can be proved in a much simpler fashion. See K. Stein, *Zur Theorie der Funktionen mehrerer komplexer Veränderlichen*, Math. Annalen, vol. 114(1937), p. 557.

[2] This is the smallest convex set containing T; it is not necessarily closed.