COVARIANT CONFIGURATIONS RELATED TO ANALYTIC CURVED SURFACES

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I. Introduction

In a study of the projective differential properties of a surface a canonical development for the equation of the surface and the geometric determination of the associated reference tetrahedron are of fundamental importance. Both of these problems were solved by Wilczynski.[1] To solve the latter problem he was led to introduce and characterize geometrically the quadric known as the canonical quadric. His method of characterizing this quadric, however, was very complicated. Bompiani[2] has offered a distinctly different characterization, and Stouffer[3] has found a simple method of locating the quadric. Green[4] obtained an expansion which serves to represent a series of canonical developments, including that obtained by Wilczynski. He used, however, Wilczynski's determination of the canonical quadric to characterize a reference tetrahedron. The author[5] presented, in a recent paper, a simple method of completing the determination of the tetrahedron associated with any one of the various canonical developments of Green, without using Wilczynski's quadric. The immediate applications of this method to the theory of surfaces prompted the author to undertake the present investigation.[6]

Let us consider a general curved surface S, referred to its asymptotic net as parametric, with the fundamental differential equations in Wilczynski's canonical form

$$y_{uu} + 2by_v + fy = 0, \qquad y_{vv} + 2a'y_u + gy = 0. \tag{1}$$

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[1] E. J. Wilczynski, *Projective geometry of curved surfaces*, (Memoirs 2-3), Transactions of the American Mathematical Society, vol. 9(1908), pp. 79–120; 293–315.

[2] E. Bompiani, *Fascio di quadriche di Darboux e normale proiettiva in un punto di una superficie*, Reale Accademia dei Lincei, Rendiconti, (6), vol. 6(1927), pp. 187–190.

[3] E. B. Stouffer, *A geometrical determination of the canonical quadric of Wilczynski*, Proceedings of the National Academy of Sciences, (18), vol. 3(1932), pp. 252–255.

[4] G. M. Green, *Memoir on the general theory of surfaces and rectilinear congruences*, Transactions of the American Mathematical Society, vol. 20(1919), pp. 79–153.

[5] P. O. Bell, *Tetrahedra associated with canonical expansions for a curved surface*, Bulletin of the American Mathematical Society, vol. 41(1935), pp. 353–355.

[6] The results of this study are presented in complete detail in the author's doctoral dissertation, University of California, 1936. The author takes this opportunity to acknowledge his indebtedness to Miss P. Sperry and to Professor E. B. Stouffer for the encouragement and many helpful suggestions which they offered during the preparation of the present paper.