THE MEASURE OF TRANSITIVE GEODESICS ON CERTAIN THREE-DIMENSIONAL MANIFOLDS

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Introduction. The problem of the existence of transitive geodesics on two-dimensional manifolds of constant negative curvature has been completely solved by Koebe [1].[1] These manifolds are obtained by assigning a hyperbolic metric to the interior of the unit circle in the complex plane and by considering as identical the points congruent under a Fuchsian group.

The question of the measure of the transitive geodesics on such manifolds has also been treated but has not been completely solved. Using the theory of continued fractions, Artin [2] proved that almost all geodesics are transitive if the group is the modular group. Myrberg [3] proved that the same result holds if the group is of the first kind (i.e., one which ceases to be properly discontinuous on the unit circle U), has a finite set of generators and has a fundamental region either lying, with its boundary, entirely inside U or having all its vertices on U. These results are included in the work of E. Hopf [4], who shows that metrical transitivity holds if the group is of the first kind with a finite set of generators. Metrical transitivity implies that almost all the geodesics are transitive. The case of an infinite set of generators has not been considered.

Three-dimensional manifolds of constant negative curvature can be obtained by assigning a hyperbolic metric

$$ds^2 = \frac{4(dx^2 + dy^2 + dz^2)}{(1 - x^2 - y^2 - z^2)^2}$$

to the interior of the unit sphere S and considering as identical the points congruent to each other under suitable groups of the rigid motions of this space. The groups must be properly discontinuous within S but may or may not be properly discontinuous on S. An example of such a manifold is the one obtained by using the Picard group. Recently Löbell [5] gave examples of closed manifolds of constant negative curvature.

As to the properties of geodesics on these manifolds Löbell [5] states that, by methods analogous to those in his proofs for two-dimensional manifolds [6], it can be shown that the periodic geodesics are everywhere dense among the totality of geodesics and that there exist transitive geodesics on the manifolds which he has set up.

It is the object of this paper to prove that, for certain three-dimensional

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[1] The numbers in brackets refer to the bibliography at the end of the paper.