

# CORRECTIONS TO “LOG ABUNDANCE THEOREM FOR THREEFOLDS”

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In Section 6 of *Log abundance theorem for threefolds* by Sean Keel, Kenji Matsuki, and James McKernan [1], there are several mistakes. Though they are minor in essence, to correct them requires more than a few changes in statements. We decided to rewrite the entire section, which is now only 6 pages long, rather than presenting a list of errata. We are extremely grateful to Qihong Xie for pointing out these mistakes.

## 6. Bogomolov stability and Mori’s bending and breaking technique

We fix some notation for this section.

Let  $X$  be a normal projective variety of dimension  $n$  over an algebraically closed field of characteristic zero. Let  $D_1, D_2, \dots, D_n$  be a sequence of nef Cartier divisors, let  $H_1, H_2, \dots, H_n$  be a sequence of  $\mathbb{Q}$ -ample divisors, and let  $H$  be an ample divisor.

Following [4], we may define the slope,  $\mu(\mathcal{F})$ , and stability of a reflexive sheaf  $\mathcal{F}$  with respect to  $D_1, D_2, \dots, D_{n-1}$ . See [4] for more details.

In the case where  $X$  has quotient singularities in codimension 2 (e.g., if  $X$  has Kawamata log terminal singularities), let  $\hat{c}_2$  be the second Chern class of the sheaf  $\hat{\Omega}_X^1$ , which is a  $Q$ -sheaf in codimension 2 (see [2, Chap. 10] and [6] for more details on  $Q$ -sheaves). In fact, although [2, Chap. 10] has the explicit assumption that  $\dim X = 2$ , the Chern classes of the  $Q$ -sheaf  $\hat{\Omega}_X^1$  are well defined in arbitrary dimension over the locus where  $X$  has quotient singularities and where the cover  $\tilde{X}$ , constructed by Mumford, is Cohen-Macaulay. Since the complement of this locus has codimension 3, a cycle of codimension at most 2 extends uniquely to a cycle on the whole of  $X$ . Thus  $\hat{c}_2$  is a well-defined cycle on  $X$ , and for similar reasons, the cycles  $c_1 = K_X$  and  $c_1^2 = K_X^2$  are also well defined. In particular, the intersection numbers

$$c_1 \cdot D_1 \cdot \dots \cdot D_{n-1}, \quad c_1^2 \cdot D_1 \cdot \dots \cdot D_{n-2}, \quad \text{and} \quad \hat{c}_2 \cdot D_1 \cdot \dots \cdot D_{n-2}$$

are certainly well defined.

The aim of this section is to prove the following.

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