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TWO-BODY SHORT-RANGE SYSTEMS IN A TIME-PERIODIC ELECTRIC FIELD

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1. Introduction. In this paper, we treat the scattering problem for two ν-dimensional particles interacting through a short-range potential and placed in an external time-periodic electric field. The Hamiltonian for such a system is

$$H(t) = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - q_1\mathscr{E}(t)\cdot x_1 - q_2\mathscr{E}(t)\cdot x_2 + v\big(x_2 - x_1\big) \quad \text{on } L^2\big(\mathbb{R}^{2\nu}\big). \tag{1.1}$$

Here, m_i and q_i, $i \in \{1,2\}$, are the masses and the charges of the two particles, and x_1, x_2 denote their positions. The electric field $\mathscr{E}$ is periodic with some period $T > 0$; that is $\mathscr{E}(t+T) = \mathscr{E}(t)$ almost everywhere. The short-range potential v will be allowed to have an explicit time-dependence as long as this dependence is periodic with the same period as the field.

Recently, asymptotic completeness for many-body systems in constant electric fields has been proved for large classes of potentials; see [AT], [HMS1], and [HMS2]. For a treatment of propagation estimates for such systems, see [A]. All these results rely on well-known techniques that use local commutator estimates to obtain spectral and scattering information. By controlling the energy along the time evolution, one can apply some of these techniques to time-dependent problems. This has been done in [SS] and has been applied in [Si2]. In [G1] and [Z], time-boundedness of the kinetic energy plays an essential role, and in [HL], the problem of bounding the kinetic energy is treated for repulsive potentials using positive commutator techniques.

In the present problem, however, one readily observes that the energy is generally not bounded in time. In fact, for very simple examples like $\mathscr{E}(t) = 1/2 + \cos(t)$ ($\nu = 1$ and $v = 0$), one finds that the expectation value of the energy oscillates with an amplitude that grows like t^2. On the other hand, the expectation of $x_2 - x_1$ grows like $(q_2/m_2 - q_1/m_1)t^2$ as one would expect from the constant field problem. Consequently, it is natural to suggest that completeness and absence of bound states (the meaning of which is discussed later) hold here as well, provided the particles have different charge to mass ratio. In order to ensure the growth of $x_2 - x_1$, we make the crucial assumption that the field has nonzero mean.

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