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AN ANALOGUE OF SERRE'S CONJECTURE FOR GALOIS REPRESENTATIONS AND HECKE EIGENCLASSES IN THE mod p COHOMOLOGY OF $\mathrm{GL}(n,\mathbb{Z})$

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1. Introduction. Let p be a prime number and $\mathbb{F}$ an algebraic closure of the finite field $\mathbb{F}_p$ with p elements. Let n and N denote positive integers, N prime to p. We are interested in representations ρ of the Galois group $G_{\mathbb{Q}} = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ into $\mathrm{GL}(n,\mathbb{F})$, unramified at all finite primes not dividing pN. (We shall say ρ is unramified outside pN.) In this paper, *representation* will always mean continuous, semisimple representation.

We choose for each prime l not dividing pN a Frobenius element Frob_l in $G_{\mathbb{Q}}$. We also fix a complex conjugation $\mathrm{Frob}_\infty \in G_{\mathbb{Q}}$. For every prime q, we fix a decomposition group G_q with its filtration by its ramification subgroups $G_{q,i}$. We denote the whole inertia group $G_{q,0}$ by I_q.

Our aim is to make a conjecture about when such a representation should be attached to a cohomology class of a congruence subgroup of level N of $\mathrm{GL}(n,\mathbb{Z})$. Then we exhibit such evidence for the conjecture as we are able.

Set $\Gamma_0(N)$ to be the subgroup of $\mathrm{SL}(n,\mathbb{Z})$ consisting of those matrices whose first row is congruent to $(*,0,\dots,0)$ modulo N. Let S_N be the subsemigroup of the integral matrices in $\mathrm{GL}(n,\mathbb{Q})$ whose first row is congruent to $(*,0,\dots,0)$ modulo N and with determinant positive and prime to N.

We denote by $\mathcal{H}(N)$ the $\mathbb{F}$-algebra of double cosets $\Gamma_0(N)S_N\Gamma_0(N)$. It is commutative. This algebra acts on the cohomology and homology of $\Gamma_0(N)$ with any coefficient $\mathbb{F}S_N$-module. When a double coset is acting on cohomology, we call it a *Hecke operator*. The Hecke algebra $\mathcal{H}(N)$ contains all double cosets of the form $\Gamma_0(N)D(l,k)\Gamma_0(N)$, where $D(l,k)$ is the diagonal matrix with k l's followed by $(n-k)$ l's, and l is a prime not dividing N. We use the notation $T(l,k)$ for the corresponding Hecke operator.

Definition 1.1. Let $\mathcal{V}$ be an $\mathcal{H}(pN)$-module, and suppose $v \in \mathcal{V}$ is an eigenvector for the action of $\mathcal{H}(pN)$ with $T(l,k)v = a(l,k)v$ for some $a(l,k) \in \mathbb{F}$, for all $k = 0,\dots,n$, and for all l prime to pN. Let ρ be a representation $\rho : G_{\mathbb{Q}} \to \mathrm{GL}(n,\mathbb{F})$

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