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REFLECTIONLESS SCHRÖDINGER OPERATORS, THE DYNAMICS OF ZEROS, AND THE SOLITONIC SATO FORMULA

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CONTENTS

1. Introduction 270
2. Reflectionless Schrödinger operators 274
2.1. The equations of motion for the zeros 274
2.2. Hamiltonian structure 277
2.3. Lax representation and integrals of motion 279
3. The action-angle transformation 282
3.1. Spectral properties of the Lax matrix 282
3.2. Linearization 285
3.3. The integral curve and action-angle diffeomorphism 288
3.4. Symplectic structure 290
4. The wave function and the tau function 292
4.1. A Wilson-type determinantal formula 292
4.2. The solitonic Sato formula 295
4.3. The Bethe curve 296
5. Dynamics and scattering of zeros 300
6. The Korteweg–de Vries hierarchy 307
Appendix A. Proof of the no-crossing Lemma 2.2 310
Appendix B. Some Cauchy matrix identities 312

The zeros of the (Jost) eigenfunction of a 1-dimensional Schrödinger operator with a reflectionless rapidly decreasing potential are related to the spectral data through a nonlinear algebraic system of Bethe-type equations. We show that the behavior of these zeros (with respect to translations) is governed by a rational Ruijsenaars-Schneider particle system with harmonic term. The integration of the particle system—via an explicit construction of the action-angle transform—then provides us with detailed information on the solution curve of the Bethe equations. As a result, we find

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