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PANEITZ-TYPE OPERATORS AND APPLICATIONS

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To the memory of André Lichnerowicz

Given (M, g) a smooth 4-dimensional Riemannian manifold, let S_g be the scalar curvature of g, and let Rc_g be the Ricci curvature of g. The Paneitz operator, discovered in [21], is the fourth-order operator defined by

$$P_g^4 u = \Delta_g^2 u - \operatorname{div}_g \left(\frac{2}{3} S_g g - 2 Rc_g \right) du,$$

where $\Delta_g u = -\operatorname{div}_g \nabla u$ is the Laplacian of u with respect to g. When (M, g) is the 4-dimensional standard unit sphere (S^4, h), we get that

$$P_h^4 u = \Delta_h^2 u + 2\Delta_h u.$$

The Paneitz operator is conformally invariant in the sense that if $\tilde{g} = e^{2\varphi} g$ is a conformal metric to g, then for all $u \in C^\infty(M)$,

$$P_{\tilde{g}}^4 u = e^{-4\varphi} P_g^4(u).$$

The 2-dimensional analogue of this relation is

$$\Delta_{\tilde{g}} u = e^{-2\varphi} \Delta_g u.$$

When the dimension is 2, it is well known that the scalar curvatures of g and $\tilde{g}$ are related by the equation

$$\Delta_g \varphi + \frac{1}{2} S_g = \frac{1}{2} S_{\tilde{g}} e^{2\varphi}.$$

When the dimension is 4, we get that

$$P_g^4 \varphi + Q_g^4 = Q_{\tilde{g}}^4 e^{4\varphi},$$

where

$$Q_g^4 = \frac{1}{6}\big(\Delta_g S_g + S_g^2 - 3|Rc_g|^2 \big).$$

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