Vol. 103, No. 2 DUKE MATHEMATICAL JOURNAL

TOPOLOGICAL ENTROPY ON SADDLE SETS IN $\mathbf{P}^2$

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0. Introduction. The Fatou set $\mathcal{F}$ of a holomorphic map $f : \mathbf{P}^k \circlearrowleft$ is the largest open subset of $\mathbf{P}^k$ on which iterates of f form a normal family. The complement of $\mathcal{F}$ is called the *Julia set* when $k = 1$, and it is well known that the Julia set is the closure of the set of repelling periodic points. When $k = 2$, however, even product maps suffice to show that the structure of $\mathbf{P}^2 \setminus \mathcal{F}$ is more intricate. For instance, $\mathbf{P}^2 \setminus \mathcal{F}$ contains both repelling periodic points and periodic points of "saddle" type, with one expanding and one contracting direction. In nice situations, these distinct types of periodic points occupy distinct regions in $\mathbf{P}^2 \setminus \mathcal{F}$, and each of these smaller regions legitimately vies with $\mathbf{P}^2 \setminus \mathcal{F}$ for the designation of Julia set.

Our concern in this paper is with what we call *saddle sets* of a holomorphic map $f : \mathbf{P}^2 \circlearrowleft$. These generalize the notion of a saddle periodic point, and while we defer the precise definition until Section 1, the following description should suffice for the moment. A closed invariant set $\Lambda = f(\Lambda) \subset \mathbf{P}^2$ is a saddle set of f if f acts transitively and hyperbolically on Λ with one contracting and one expanding direction, and if Λ is in some sense both maximal and isolated as a hyperbolic set. Important examples of saddle sets are given by the basic sets of saddle type for an Axiom A map $f : \mathbf{P}^2 \circlearrowleft$. Indeed, the paper [FS2] of Fornæss and Sibony on Axiom A holomorphic maps of $\mathbf{P}^2$ inspired much of the work on which this paper is based.

Given a history $\hat{p} = (p_j)_{j \le 0}$ in Λ (i.e., $p_j \in \Lambda$ and $f(p_{j-1}) = p_j$ for all $j \le 0$) and a small fixed $\delta > 0$, the associated local unstable manifold is

$$W^u_{\mathrm{loc}}(\hat{p}) = \left\{ q \in \mathbf{P}^2 : \operatorname{dist}(q_j, p_j) < \delta \text{ for } j \le 0 \text{ and some } \hat{q} \text{ with } q_0 = q \right\}.$$

As is well known, f expands local unstable manifolds near Λ. Nevertheless, we call Λ terminal if for each history $\hat{p}$ in Λ, iterates of f act normally on $W^u_{\mathrm{loc}}(\hat{p}) - \Lambda$. We believe that terminal saddle sets play a distinguished role in the global dynamics of f. Our main result is the following theorem.

THEOREM A. *Let Λ be a saddle set of a holomorphic map $f : \mathbf{P}^2 \circlearrowleft$ of degree d. Then the topological entropy $h_{\mathrm{top}}(f|_\Lambda)$ of f restricted to Λ is no greater than* $\log d$. *Equality holds if and only if Λ is terminal.*

Received 19 July 1999.

2000 *Mathematics Subject Classification*. Primary 32H50; Secondary 37F15, 37D05, 37D20.

Diller supported by National Science Foundation grant number DMS98-96370.

Jonsson supported by the Swedish Foundation for International Cooperation in Research and Higher Education (STINT).