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EQUIVARIANT K-THEORY, WREATH PRODUCTS, AND HEISENBERG ALGEBRA

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0. Introduction. Given a finite group G and a locally compact, Hausdorff, paracompact G-space X, the nth direct product X^n admits a natural action of the wreath product $G_n = G \sim S_n$, which is a semidirect product of the nth direct product G^n of G and the symmetric group S_n. The main goal of the present paper is to study the equivariant topological K-theory $K_{G_n}(X^n)$ for all n together, and discuss several applications that are of independent interest.

We first show that a direct sum

$$\mathcal{F}_G(X) = \bigoplus_{n \geq 0} K_{G_n}(X^n) \bigotimes \mathbb{C}$$

carries several wonderful structures. More explicitly, we show that $\mathcal{F}_G(X)$ admits a natural Hopf algebra structure with a certain induction functor as multiplication and a certain restriction functor as comultiplication (cf. Theorem 2). When X is a point, $K_{G_n}(X^n)$ is the Grothendieck ring $R(G_n)$, and we recover the standard Hopf algebra structure of $\bigoplus_{n\geq 0} R(G_n)$ (cf., e.g., [M2], [M3], [Z]). A key lemma used here is a straightforward generalization to equivariant K-theory of a statement in the representation theory of finite groups concerning the restriction of an induced representation to a subgroup.

We show that $\mathcal{F}_G(X)$ is a free λ-ring generated by $K_G(X) \bigotimes \mathbb{C}$ (cf. Proposition 3). We write down explicitly the Adams operations φ^n's in $\mathcal{F}_G(X)$. Incidentally, we also obtain an equivalent way of defining the Adams operations in $K_G(X) \bigotimes \mathbb{C}$ (not over $\mathbb{Z}$) by means of the wreath products, generalizing a definition by Atiyah [A1] in terms of the symmetric group in the ordinary (i.e., nonequivariant) K-theory setting. When X is a point, we recover the λ-ring structure of $\bigoplus_{n\geq 0} R(G_n)$ (cf. [M2]).

As a graded algebra, $\mathcal{F}_G(X)$ has a simple description as a certain supersymmetric algebra in terms of $K_G(X) \bigotimes \mathbb{C}$ (cf. Theorem 3). The proof uses a theorem in [AS] and the structures of the centralizer group of an element in G_n and of the fixed-point set of the action of $a \in G_n$ on X^n, which we work out in Section 1. In particular, this description indicates that $\mathcal{F}_G(X)$ has the size of a Fock space of a certain infinite-dimensional Heisenberg superalgebra that we construct in terms of natural additive maps in K-theory (cf. Theorem 4).

Our results above generalize Segal's work [S2], and our proofs are direct generalizations of those in [S2] (also see [Z], [M1]). What Segal studied in [S2], partly

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