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RIEMANN-ROCH FOR EQUIVARIANT CHOW GROUPS

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1. Introduction. The purpose of this paper is to prove an equivariant Riemann-Roch theorem for schemes or algebraic spaces with an action of a linear algebraic group G. For a G-space X, this theorem gives an isomorphism

$$\tau^G : G^G(X) \longrightarrow \widehat{G^G(X)}_{\mathbb{Q}} \xrightarrow{\simeq} \prod_{i=0}^{\infty} CH^i_G(X)_{\mathbb{Q}}.$$

Here $\widehat{G^G(X)}$ is the completion of the equivariant Grothendieck group of coherent sheaves along the augmentation ideal of the representation ring $R(G)$, and the groups $CH^i_G(X)$ are the equivariant Chow groups defined in [EG2]. The map τ^G has the same functorial properties as the nonequivariant Riemann-Roch map of [BFM] and [F, Theorem 18.3]. If G acts freely, then τ^G can be identified with the nonequivariant Todd class map $\tau_{X/G} : G(X/G) \to CH^*(X/G)_{\mathbb{Q}}$.

The key to proving this isomorphism is a geometric description of completions of the equivariant Grothendieck group (see Theorem 2.1). Aside from Riemann-Roch, this result has some purely K-theoretic applications. In particular, we prove (see Corollary 6.2) a conjecture of Köck (in the case of regular schemes over fields) and extend to arbitrary characteristic a result of Segal on representation rings (see Corollary 6.1).

For actions with finite stabilizers, the equivariant Riemann-Roch theorem is more precise; it gives an isomorphism between a localization of $G^G(X)_{\mathbb{Q}}$ and $\oplus CH^i_G(X)_{\mathbb{Q}}$ (see Corollary 5.1). This formulation enables us to give a simple proof of a conjecture of Vistoli (see Corollary 5.2). If G is diagonalizable, then we can express $G^G(X)$ in terms of the equivariant Chow groups (an unpublished result of Vistoli; cf. [To] also). Actions with finite stabilizers are particularly important because quotients by these actions arise naturally in geometric invariant theory. In a subsequent paper, we will use these results to express the Todd class map for a quotient of such an action in terms of equivariant Todd class maps, generalizing Riemann-Roch formulas of Atiyah and Kawasaki.

The main tool of this paper is the approximation of the total space of the classifying bundle EG by an open subset U of a representation V, where G acts freely on U and where $V - U$ is a finite union of *linear* subspaces. Approximations to EG by open sets

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