HIGHER INTEGRABILITY FOR PARABOLIC SYSTEMS OF p-LAPLACIAN TYPE

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1. Introduction. In this work, we study regularity of solutions to second-order parabolic systems:

$$\frac{\partial u_i}{\partial t} = \operatorname{div} A_i(x,t,\nabla u) + B_i(x,t,\nabla u), \quad i = 1,\dots,N. \tag{1.1}$$

In particular, we are interested in systems of p-Laplacian type. We present more precise structural assumptions later, but the principal prototype that we have in mind is the p-parabolic system

$$\frac{\partial u_i}{\partial t} = \operatorname{div}\big(|\nabla u|^{p-2}\nabla u_i\big), \quad i = 1,\dots,N,$$

with $1 < p < \infty$. As usual, solutions to (1.1) are taken in a weak sense, and they are assumed to belong to a parabolic Sobolev space. A good source for the regularity theory is [D].

In the elliptic case when the system is

$$\operatorname{div} A_i(x,t,\nabla u) + B_i(x,t,\nabla u) = 0, \quad i = 1,\dots,N, \tag{1.2}$$

it is known that solutions locally belong to a slightly higher Sobolev space than assumed a priori. This self-improving property was first observed by Elcrat and Meyers in [ME] (see also [Gi] and [Str]). Their argument is based on reverse Hölder inequalities and a modification of Gehring's lemma [Ge], which originally was developed to study the higher integrability of the Jacobian of a quasiconformal mapping. In the elliptic case, higher integrabilty results play a decisive role in studying the regularity of solutions (see [GM] and [Gi]).

The purpose of this work is to obtain higher integrablity results in the p-parabolic setting. We prove that the gradient of a weak solution to (1.1) satisfies a reverse Hölder inequality for $p > 2n/(n+2)$. The critical exponent $2n/(n+2)$ occurs also in parabolic regularity theory (see [D]). We note that reverse Hölder inequalities and the local higher integrability for weak solutions were already proved for $p = 2$ in [GS] (see also [C]). Our result appears to be new even in the scalar case if $p \neq 2$.

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