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SOLUTIONS, SPECTRUM, AND DYNAMICS FOR SCHRÖDINGER OPERATORS ON INFINITE DOMAINS

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1. Introduction and main results. In this paper we investigate the relations between the rate of decay of solutions of Schrödinger equations, continuity properties of spectral measures of the corresponding operators, and dynamical properties of the corresponding quantum systems. The first main result of this paper shows that, in great generality, certain upper bounds on the rate of growth of L^2 norms of generalized eigenfunctions over expanding balls imply certain minimal singularity of the spectral measures. Consider an operator H_V^Ω defined by the differential expression

$$H_V^\Omega = -\Delta + V(x)$$

on some connected infinite domain Ω with a smooth boundary and with Dirichlet boundary conditions on $\partial\Omega$. The case of $\Omega = \mathbb{R}^d$ is not excluded; no boundary conditions are needed in this case. To every vector $\phi \in L^2(\Omega)$ we associate a spectral measure μ^ϕ in the usual way (namely, μ^ϕ is the unique Borel measure on $\mathbb{R}$ obeying $\int f(E)\,d\mu^\phi(E) = (f(H_V^\Omega)\phi, \phi)$ for any Borel function f). For any measure μ, we define the upper α-derivative $D^\alpha \mu(E)$ in the standard way:

$$D^\alpha \mu(E) = \limsup_{\delta\to 0} \frac{\mu(E-\delta, E+\delta)}{\delta^\alpha}.$$

We denote by B_R the ball of radius R centered at the origin, and we use the notation $\|f\|_{B_R}$ for the L^2 norm of the function f restricted to B_R. We denote by W_m^l the usual Sobolev spaces of functions f such that $D^l f$ exists in the distributional sense and $\int (|u|^m + |D^l u|^m)\,dx < \infty$. We say that $f(x) \in W_{m,\mathrm{loc}}^l(\Omega)$ if $f(x) \in W_m^l(\Omega \cap B_R)$ for every $R < \infty$. One of the main theorems that we prove here is the following.

THEOREM 1.1. *Assume that the potential $V(x)$ belongs to L^∞_{loc} and is bounded from below, and that Ω is a domain with piecewise smooth boundary. Suppose that there exists a distributional solution $u(x,E)$ of the generalized eigenfunction equation*

$$\big(H_V^\Omega - E\big)u(x,E) = 0 \tag{1}$$

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