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SINGULAR POINTS AND LIMIT CYCLES OF PLANAR POLYNOMIAL VECTOR FIELDS

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CONTENTS

0. Introduction. The subject of the article is real planar vector fields $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where P, Q are polynomials. We consider two questions related to the second part of the Hilbert 16th problem [Hi].

(1) What can be the number and arrangement of limit cycles of a vector field of degree d in $\mathbb{R}^2$?

(2) Given a classification of singular points of planar vector fields, how many singular points of each type can a vector field of degree d in $\mathbb{R}^2$ have?

Our approach to these problems comes from the Viro method [V1] to [V4] (see also [IV] and [R]) invented in the framework of the first part of the 16th problem, topology of real algebraic varieties. This method, actually, consists in reducing a problem on polynomials with an arbitrary Newton polyhedron to that on polynomials with smaller Newton polyhedra. Various applications and developments related to the topology of real algebraic varieties and their singularities can be found in [GKZ], [I1], [I2], [IV], [S1] to [S3], and [St]. Note also that Newton polyhedra and diagrams have been used since the last century for the local study of singular points of differential systems. (For the modern account, see, e.g., [Br].)

In this paper we prove Viro-type "gluing" theorems for planar polynomial vector fields (see Theorems 1.3.1, 1.4.1, and Corollary 1.4.2). Using gluing theorems we construct vector fields with many limit cycles and vector fields with given numbers of singular points of prescribed types. The exact statements (see Theorem 2.1 on limit

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