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SINGLY GENERATED PLANAR ALGEBRAS OF SMALL DIMENSION

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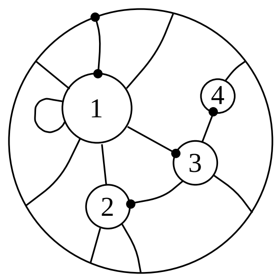
0. Introduction. A subfactor $N \subset M$ gives rise to a powerful set of invariants that can be approached successfully in several ways. (See, for instance, [B2], [EK], [FRS], [GHaJ], [H], [Iz], [JSu], [Lo1], [Lo2], [Oc1], [Oc2], [Po1], [Po2], [Po3], [Po4], [Wa], [We1], and [We2]). A particular approach suggests a particular kind of subfactor as the "simplest." For instance, in Haagerup's approach [H], subfactors of small index are the simplest. In [J2], a pictorial language is developed in which the invariants appear as a graded vector space $V = (V_n)_{n \geq 0}$ whose elements can be combined in planar, but otherwise quite arbitrary, ways. Thus, for instance, in the diagram



every time one assigns elements $v_1 \in V_4$, $v_2 \in V_2$, $v_3 \in V_3$, and $v_4 \in V_1$ to the "empty boxes" 1, 2, 3, and 4, there is associated, in a multilinear and natural way, an element of V_4. The grading of the V_n's is given by half the number of strings attached to the boundary of the box. Various algebra and other structures are given by particular planar ways of combining elements. It is shown in [J2] that under appropriate positivity conditions on V (summed up by saying that V is a *subfactor planar algebra*), there is a subfactor $N \subset M$ having V_n as its higher relative commutant $N' \cap M_{n-1}$.

Any subset S of V then generates a planar subalgebra as the smallest graded vector space containing S and closed under planar operations. From this point of view, the simplest subfactors are those whose planar algebra is generated by the smallest sets S,

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