Comment

Persi Diaconis and Eduardo Engel

I. J. Good has given us a marvelous historical perspective, blending the subjective/objective dichotomy with the mathematics of Poisson's summation formula. These subjects are intimately related to recent work on probability in classical physics. We first develop this connection, and then compare Poisson's techniques with Poincaré's method of arbitrary functions. We conclude by outlining generalizations connected to Selberg's trace formula.

1. A SUBJECTIVE GUIDE TO OBJECTIVE CHANCE

Physics and mathematics offer a useful way of relating subjective probability to objective chance devices. Consider throwing a real dart at a real wall. If the left half of the wall is painted black, and the right half painted white, there is nothing very random about the outcome: by aiming a bit to the left, the dart winds up in the black section.

Now suppose the paint is rearranged to form stripes which are alternately painted black and white. If the distance between the stripes is large, things still aren't random, but as the distance gets smaller, black and white will be judged nearly equally likely by almost anyone.

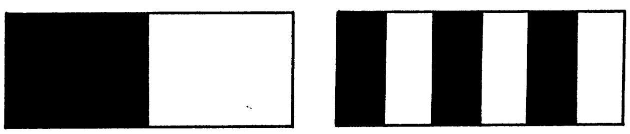

FIG. 1

It is not difficult to give quite sharp quantitative estimates: Suppose the thrower stands a distance l from the wall, and the stripes have width d. Clearly only the ratio l/d matters, so without loss of generality, take $l = 1$. The situation is pictured in Figure 2.

Let θ be the angle of release of the dart and suppose $f(\theta)$ is a probability density on $(-\pi/2, \pi/2)$. If θ is chosen from f,

$$P\{\text{Black}\} = \sum_{n=-\infty}^{+\infty} \int_{\theta_{2n}}^{\theta_{2n+1}} f(\theta)\, d\theta \tag{1.1}$$

with

$$\theta_n = \tan^{-1}\left(\frac{1}{nd}\right).$$

A result due to Kemperman (1975) (discussed in Section 2) and straightforward calculus lead to the bound

$$|P\{\text{Black}\} - \tfrac{1}{2}| \le cd$$

with

$$c = \frac{V(f) + 1}{4} \quad \text{and} \quad V(f) = \int_{-\pi/2}^{\pi/2} |f'(\theta)|\, d\theta.$$

Now let us consider the philosophical implications of the mathematics. If $f(\theta)$ is Smith's subjective distribution of the angle of release, and if $f(\theta)$ is not too sharply peaked, then Smith is forced to assign probability about ½ to the dart landing in the black region.

This gives an objective chance device in the following sense: Most people will assign the same probability to the outcome black even though they may have very different prior beliefs about θ.

This is quite different from the usual argument for multisubjective agreement ("the data swamp the prior"). It applies to a single event: Agreement is reached without the need of any data.

A similar analysis can be given for many other objective chance devices. Consider flipping a coin in the air and catching it as it lands. When the coin leaves the hand, its upward velocity and angular momentum completely determine which side will land uppermost. It is possible to carry out the physics and

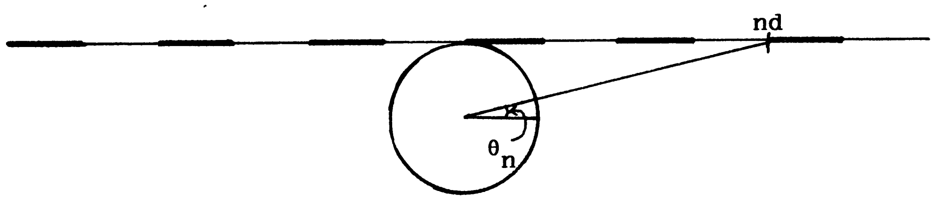

FIG. 2

Persi Diaconis is Professor of Statistics and Eduardo Engel is a graduate student in the Department of Statistics, Stanford University, Sequoia Hall, Stanford, California 94305.