and processing only via probabilistic means (e.g., Bayes theorem), while explicitly recognizing the inexactness of probability elicitation. This approach, long advocated by I. J. Good (cf. Good, 1983), is reviewed and discussed (as the "robust Bayesian" viewpoint) in Berger (1984, 1985). Of particular note, in terms of axiomatics, is that Smith (1961), Good (1962), Giron and Rios (1980), and others show that possible noncomparability, together with a reasonable set of other axioms, essentially yield the robust Bayesian approach.

As a second example of how "reality" might impact on axiomatics, consider the issue of finitely additive versus countably additive probabilities. Axiomatically, additional assumptions must be made to guarantee countably additive probabilities, assumptions which tend to be somewhat obscure and noncompelling. Attempts to work with finitely additive probabilities, however, encounter the difficulty that conditional distributions (or posterior probabilities) are often not well-defined, so that additional assumptions end up being needed anyway. And the nature of these assumptions is perhaps even more obscure than those leading to countable additivity; one might well conclude that the countably additive domain is the least objectionable arena in which to perform.

ACKNOWLEDGMENT

This research was supported by the National Science Foundation under Grant DMS-8401996.

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Comment

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My remarks focus on the themes of extension, tolerance for limited precision, the restricted applicability of the familiar concept of numerical probability, and the possibilities for other concepts of probability that are suggested by the axiomatic measurement-theoretic approach to comparative probability. Dr. Fishburn provides us with an authoritative survey of several axiom systems for binary relations of comparative (qualitative) probability that have been developed in the context of an interpretation of subjective probability based upon the degrees of belief of an individual. One might hope that a study of such axiom systems for comparative probability would lead us closer to the conceptual issues and roots of probabilistic reasoning and rational beliefs about uncertainty and thereby also enable us to develop such reasoning processes and model such beliefs through a probability-like mathematical structure. A process of axiomatization enables us to decompose a complex issue into a related set of simpler component issues that can then be examined closely on their merits. When properly engaged in, such a study does not prejudge its outcome. By observing the nature and strength of the axioms necessary to insure that the resulting model is a finitely or countably additive numerical probability measure, we can gain insight into the limitations of this familiar and often reliable model. By eliminating those axioms that appear to be objectionable in particular application domains, we can develop alternative concepts of probability useful for fairly representing probabilistic reasoning about either individual beliefs or objective nondeterministic phenomena, as appropriate for the domain. Clearly, the process of axiom selection must be guided by sound interpretations of the probability concept.

Regrettably, but understandably, few of these issues are addressed with sufficient emphasis either in this survey or in much of the related literature cited therein. While the opening quotations might lead us to anticipate an analysis of the link between belief or expectation (on the subjective interpretation) and the mathematical apparatus that is then deployed, this is

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