flat regions with occasional spikes. It might be useful to develop smoothing functionals J that mirror this.

Regarding the empirical selection of smoothing parameters, Rice (1986) sounds a cautionary note by constructing simple examples in which a choice of smoothing parameter giving a good value of predictive mean square error gives unacceptable errors for estimating θ and vice versa.

ADDITIONAL REFERENCES

RICE, J. (1986). Choice of smoothing parameter in deconvolution problems. *Contemp. Math.* **59** 137–151.

WAHBA, G. (1982). Constrained regularization of ill-posed linear operator equations, with applications in meteorology and medicine. In *Statistical Decision Theory and Related Topics III.* (S. Gupta and J. Berger, eds.) **2** 383–418. Academic, New York.

Comment

Freeman Gilbert

In a typical geophysical inverse problem one has

$$(1) \qquad d_j = D_j(f) + r_j\sigma_j, \qquad j \in \{1, \ldots, J\},$$

where

- d is a datum,
- D is the functional that maps f into d,
- f is the model,
- r is a unit variance random variable,
- σ is the assigned error, usually taken to be the standard deviation (Gaussian errors).

An error statistic is introduced, usually the χ^2 statistic

$$(2) \qquad \chi^2(f) = \sum_j [d_j - D_j(f)]^2/\sigma_j.$$

One defines the set

$$\{F_0(f)\text{: all } f \text{ such that } \chi^2(f) \le X_0^2\},$$

where χ_0^2 is chosen to be the 99% or 95% confidence level, for example.

Except in very unusual circumstances, $F_0(f)$ is either empty or infinite-dimensional. In the former case, one increases χ_0^2 and seeks to fill $F_0(f)$. In the latter case, one desires to know about the members of $F_0(f)$.

One procedure is to use the method of regularization (MOR) to find a particular member of $F_0(f)$ (e.g., the smallest, the smoothest, the one closest to a particular f_0, the maximum entropy solution, $\max\{-f\log f\}$, etc.). Another procedure is to use a resolution method to find what features all f have in common or what are the resolvable averages of f. In any case one may wish to assert a priori conditions on f, such as prejudices about the shape or size of f that can be cast in the form of equation (1).

O'Sullivan has shown that the two procedures are connected and, taken together, can lead to improved methods of estimating bias. By generalizing the concept of averaging kernel, i.e., requiring the averaging kernel to assume certain shapes, one can estimate average bias as well as local bias. For linear problems, the matter appears to be resolved and depends only on the number and quality of the data and the span of their representers. For nonlinear problems one is confined to the neighborhood of the subject. O'Sullivan is to be congratulated for his original contribution to it.

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Rejoinder

Finbarr O'Sullivan

It is a pleasure to thank the discussants Professors Gilbert, Rice, Titterington, and Wahba for their most interesting and stimulating comments. The ubiquity of inverse problems in areas like geophysics, medical imaging, and meteorology presents statisticians with wonderful opportunities to contribute to the development of science and technology. As Professor Wahba notes there are lots of open research questions many