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Rejoinder

C. Radhakrishna Rao

For ready reference, the problem considered in the paper is the following. We have observations (U_i, W_i), where U_i is a p vector of measurements taken at p time points and W_i is the measurement taken at a future $(p+1)$th time point, on $i = 1, \cdots, n$ individuals drawn from a population S. Another individual drawn from S provides the first p measurements U_c, and the problem is to predict the $(p+1)$th measurement W_c on the individual.

What is relevant in a problem of this kind is the conditional (predictive) distribution of W_c given U_c,

$$(1) \qquad P_{\text{pred}}(W_c \mid U_c, \psi),$$

with respect to some *reference* population, where ψ is a parameter specific to the reference population. One choice of the reference population is S itself. However, when ψ is unknown, we have two possibilities. We may estimate ψ by $\hat{\psi}$ from the available data

$$(2) \qquad (U_i, W_i), \quad i = 1, \cdots, n, \quad \text{and} \quad U_c$$

and consider an estimate of (1),

$$(3) \qquad P_{\text{empred}}(W_c \mid U_c, \hat{\psi}),$$

as the basic conditional distribution. An alternative is to consider S as a member of a super population generated by a prior distribution on ψ, in which case the relevant distribution is

$$(4) \qquad P_{\text{Baypred}}(W_c \mid U_c)$$

obtained by integrating (1) with respect to the posterior distribution of ψ given the observed data (2). On the other hand, we may wish to consider the current individual's observations (U_c, W_c) as arising from a stochastic process *specific* to the individual. In such a case the empred (3) is defined in terms of $\hat{\psi}$ estimated from U_c alone and the Baypred (4) is obtained by choosing a prior on ψ and computing the posterior distribution based on U_c alone. The second possibility of considering an individual separately is specially recommended when on the basis of an initial examination of data, the measurements U_c are found to have an unusual pattern different from those of $U_1, \cdots, U_n$.

The theory as developed in Section 2 of the paper and outlined above is complete in itself although its practical applications involves various issues that I would like to discuss on the basis of the comments made by the discussants of my paper.

DATA AND CROSS-EXAMINATION OF DATA

For illustrative purposes I have chosen three real data sets, which are well documented and which have been studied by a number of authors for predictive purposes. I thank Izenman for giving some details about the mice data that will be helpful to future investigators. I have made the necessary corrections regarding the original source of the dental data based on his comments. In my analysis of the mice data, I omitted the measurements on one mouse (not reported in Table 2, but can be found in Izenman's comments), which looked different from the others and whose weight actually decreased at the end. Izenman asks what effect it would have had on my results if this mouse had been retained in the data set. I have deliberately chosen my reference population as the set of mice that generally exhibit an increase in growth at all time points and derived the appropriate prediction