Comment

Ann F. S. Mitchell

Without doubt many interesting developments and applications can be expected to arise from this elegant and clear exposition of saddlepoint methods in statistics.

I have little to add except to comment on the similarities and dissimilarities in the geometry of exponential families and transformation models. To achieve the level of clarity of Nancy Reid's presentation would require a lengthy and relatively technical introduction to the basic ideas and concepts of the differential geometry approach to statistics. As a compromise, albeit a relatively unsatisfactory one, the reader is referred to Amari (1985) and Barndorff-Nielsen, Cox and Reid (1986).

Although the exponential families are flat with respect to the ± 1 connections of the family of α-connections described by Chentsov (1972), Dawid (1975, 1977) and Amari (1982), transformation models have constant scalar curvature but are not necessarily flat for any α-value. The following well known two-parameter transformation models, which are not exponential families, illustrate what can happen. The parameters are the location and scale parameters. The Cauchy family is particularly unusual. It has constant negative scalar curvature, which does not depend on α and is thus never flat. However, the corresponding Student's t family on k ($k > 1$) degrees of freedom is flat when

$$\alpha = \pm\frac{k+5}{k-1}.$$

In the limit, as $k \to \infty$, we get $\alpha = \pm 1$, the appropriate values for a normal family that is both an exponential family and a transformation model. The logistic family is not flat for any value of α. These examples are special cases of a subclass of transformation models, studied by Mitchell (1988), and are the univariate analogue of the well known class of multivariate elliptic distributions.

Flatness plays an important role in defining unique estimators based on projections and is discussed extensively by Amari (1985), Lauritzen and Picard (1987) and Lauritzen (1987).

Ann F. S. Mitchell is Senior Lecturer, Department of Mathematics, Imperial College, Huxley Building, 180 Queen's Gate, London SW7 2BZ, United Kingdom.

ADDITIONAL REFERENCES

BARNDORFF-NIELSEN, O. E., COX, D. R. and REID, N. (1986). The role of differential geometry in statistical theory. *Internat. Statist. Rev.* **54** 83–96.

CHENTSOV, N. N. (1972). *Statistical Decision Rules and Optimal Inference* (in Russian). Nauka, Moscow. English translation (1982). *Translation of Mathematical Monographs* **53**. Amer. Math. Soc., Providence, R. I.

DAWID, A. P. (1975). Discussion of "Defining the curvature of a statistical problem (with applications to second order efficiency)" by B. Efron. *Ann. Statist.* **3** 1231–1234.

DAWID, A. P. (1977). Further comments on some comments on a paper by Bradley Efron. *Ann. Statist.* **5** 1249.

LAURITZEN, S. L. (1987). Conjugate connections in statistical theory. *Proc. of the Geometrization of Statistical Theory*. Dept. Mathematics, Univ. Lancaster, 28–31 October.

LAURITZEN, S. L. and PICARD, D. (1987). On the uniqueness of projections in statistical manifolds. Unpublished.

MITCHELL, A. F. S. (1988). Statistical manifolds of univariate elliptic distributions. *Internat. Statist. Rev.* **56** 1–16.

Rejoinder

N. Reid

I would like to thank all the discussants for their contributions. Many of the discussants, and several colleagues, also sent me helpful comments on earlier versions of the paper.

It is a measure of the interest in and potential of saddlepoint methods that I expect this review will soon be out of date!

I am grateful to Hougaard and to Hinkley and Wang for providing additional numerical work to supplement Figure 1. Srivastava and Yau (1987) have also derived the saddlepoint expansion for the noncentral χ^2 distribution. In work as yet unpublished, Yau has derived the saddlepoint approximation for the density and Lugannani and Rice's tail area approximation for