pertaining to the robustness with respect to loss functions and distributions, of the results on estimation in the present paper.

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Comment

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I would like to begin by congratulating Maatta and Casella for an extraordinarily lucid and thought-provoking account of developments in decision-theoretic variance estimation. By systematically organizing so many related results, they have successfully exposed the main thread of ideas running through these developments. Effectively, this paper will serve as a springboard for further research ideas. To emphasize this point, my comments will focus on two new directions along which such ideas might proceed. The first concerns multiple shrinkage generalizations, and the second concerns further improvements to shrinkage estimators of the mean.

Let me mention before going on that, although my comments are limited to suggestions for future developments in point estimation, I am optimistic that these may also lead to analogous developments in interval estimation. I say this in light of the close connections between developments in these two areas which is brought out so clearly by Maatta and Casella.

1. MULTIPLE SHRINKAGE GENERALIZATIONS

A key idea behind the improved variance estimators described by Maatta and Casella is that of adaptively pooling possibly related information. In the single sample setting $X_1, \cdots, X_n \sim$ iid $N(\mu, \sigma^2)$, the estimators of Stein, Brown and Brewster and Zidek each improve on the "straw man" estimator $S^2/(n+1)$, $(S^2 = \sum (X_i - \bar{X})^2)$, by exploiting the possibility that $\mu/\sigma \approx 0$. The improved estimators are of the form $\phi(Z)S^2$, $(Z = \sqrt{n}\bar{X}/S)$, where $\phi(Z)$ is bounded above by $1/(n+1)$ and decreases as Z^2 decreases. When Z^2 is small, which is likely when μ^2/σ^2 is small, these estimators "shrink" $S^2/(n+1)$, effectively regaining the lost degree of freedom used in estimating μ. Indeed, Stein's estimator replaces $S^2/(n+1)$ by $\sum X_i^2/(n+2)$, an appropriate estimator when it is known that $\mu = 0$.

At first glance, this phenomenon may seem to be only a mathematical curiosity. After all, one degree of freedom will usually be a minor practical gain. This is precisely the point of the 4% bound on relative improvement described by Rukhin (1987a). However, it is straightforward to generalize these results to the general linear model case, as Maatta and Casella indicate in Section 5, where there are many more degrees of freedom and important gains may be realized. Indeed, the seminal results of Stein (1964) are obtained in such a case, although he states that "even in this case . . . the improvement is likely to be slight."

Unfortunately, there may be good reason to agree with Stein's pessimism. This can be seen in the canonical context of Section 5 where we observe independent normal variables $X_1, \cdots, X_\nu, X_{\nu+1}, \cdots, X_{\nu+p}$,

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