## Kolmogorov as I Remember Him

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## 1. EARLY YEARS

Andrei Nikolaevitch Kolmogorov was born in 1903 during a journey from the Crimea to his mother's home. He was the son of parents not formally married. His mother, Mariya Yakovlevna Kolmogorova, died in childbirth, and her son was adopted and brought up in the village of Tunoshna (near to Yaroslavl on the river Volga) by her sister, Vera Yakovlevna Kolmogorova. To her nephew Vera Yakovlevna gave the love of a mother, and Andrei Nikolaevitch responded with the love of a son. It is warming to be able to record that she lived until 1950, and so was able to witness some of his greatest achievements.

Andrei Nikolaevitch is always known to us by the family name of his maternal grandfather Yakov Stepanovitch Kolmogorov, and it was in the Kolmogorov family home at Tunoshna that he spent his earliest years. During his childhood the family home housed a clandestine printing press, and family traditions record that compromising documents were sometimes hidden under his cradle.

Of Kolmogorov's father, Nikolai Kataev, we know that he became a professionally trained agriculturalist, that he was exiled to Yaroslavl, that after the Revolution he became a department head in the Agriculture Ministry and that he perished on the southern front during the offensive by Denikin in 1919.

Kolmogorov went to Moscow in 1920 as a student of mathematics, but he also attended lectures in metallurgy. In addition to this he took part in a seminar on Russian history, where he presented the results of his first piece of research, on "Landholding in Novgorod in the 15th and 16th centuries."

We are told how this was received by his professor: "You have supplied one proof of your thesis, and in mathematics this would perhaps suffice, but we historians prefer to have ten proofs." This anecdote is usually told as a joke, but to those who

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A number of mathematicians stimulated Kolmogorov's earliest mathematical research, but perhaps his principal teacher was Stepanov. In 1922 Kolmogorov produced a synthesis of the French and Russian work on the descriptive theory of sets of points, and at about the same time he was introduced to Fourier series in Stepanov's seminar. This was when he made his first mathematical discovery—that there is no such thing as a slowest possible rate of convergence to zero for the Fourier cosine coefficients of an integrable function.

Nearly 30 years ago I gave a lecture in Tbilisi in which I proved that in the transient aperiodic case the diagonal Markov transition probabilities  $p_{ii}^{(n)}$  for fixed i always form a sequence of Fourier cosine coefficients, and I remarked that it would be interesting to see what one could deduce from this fact concerning the rate of convergence to zero as n tends to infinity. Kolmogorov made a comment that I did not understand at the time because of language difficulties. It occurs to me now that he must have thinking of an application of his own first paper in this new context. Thus one can ask whether the sequence of such diagonal Markov transition probabilities also has no slowest rate of convergence to zero. I do not know the answer to that question.

Figure 1 shows Kolmogorov (wearing spectacles and leaning over to his left) at the Tbilisi meeting. Also in the picture are Dynkin and Gnedenko (to Kolmogorov's right) and many other well-known probabilists. As I don't myself appear in this photograph of the audience, I like to think that it may have been my lecture they were listening to! Recently I attended a lecture by Professor Gell-Mann in which he showed a slide of himself lecturing to an audience that included Dirac, fast asleep in the front row. So I hope you will notice that in this picture Kolmogorov is still awake—if slightly worried.

Also in 1922 Kolmogorov constructed the first example of an integrable function whose Fourier series diverges almost everywhere. He was only 19 years old at the time, and suddenly he had become an international celebrity—the more so after he