this clear, I will express the same recursions in network terms.

The network consists of nodes and arcs over $k + 1$ stages labeled $0, 1, \ldots, k$. The nodes at stage j are ordered pairs of the form (j, m_j), where $m_j \in \Lambda_j$. The set $S(j, m_j)$ defines the successor nodes to the node (j, m_j), while the set $P(j, m_j)$ defines its parent nodes. If we start with an initial node $(0, 0)$ at stage 0, and apply (2.1) systematically to all the nodes created at each stage, we end up with a single terminal node (k, m_k) at stage k. Each successor to node (j, m_j) is a node of the form $(j + 1, m_{j+1})$. It is connected to (j, m_j) by an arc of length, $w_{j+1}(m_{j+1} - m_j)$ and probability $n_{j+1}!/(m_{j+1} - m_j)!(n_{j+1} - m_{j+1} + m_j)!$. A path through the network is a sequency of directed arcs connecting the initial node $(0, 0)$ to the terminal node (k, m_k). Its length is the sum of lengths, and its probability the product of probabilities, of the arcs constituting the path. Through this specification, each path through the network represents one and only one table $\mathbf{x} \in \Gamma$. Its length is given by (1.1) and its probability is given by (1.2). The problem of generating the distribution (1.6) is now equivalent to generating the distribution of the lengths of all paths through the network. The set $\Omega(j, m_j)$ represents the distribution of the lengths of all the paths from node $(0, 0)$ to node (j, m_j). The recursions (2.4) and (2.5) amount to expressing the distribution of lengths at node (j, m_j) in terms of the distributions of lengths at its parent nodes $P(j, m_j)$. Also, computing $SP(j, m_j)$ and $LP(j, m_j)$ amounts to computing the lengths of the shortest and longest paths, respectively, from node (j, m_j) to the terminal node (k, m_k). These may be obtained by backward induction on the network (Mehta, Patel and Senchaudhuri, 1992) or by more formal integer programming theorems (Joe, 1988; Agresti, Mehta and Patel, 1991).

In our research papers, although not in this discussion, the network representation of a computational problem has always preceded its algebraic representation. It is certainly elegant to express the computational problem directly in terms of recursions like (2.4) and (2.5). However, it is not so easy to gain the necessary insight to write out the recursions in the first place. Nor is it clear how one implements them on a computer once they are written down. We regard the network approach as a general technique for deriving these recursions, guiding us in selecting appropriate data structures for computer implementation, and solving the necessary integer programming problems. We have used this approach for $2 \times k$ tables, stratified $2 \times k$ tables, $r \times c$ tables and logistic regression.

In summary, this discussion has attempted to show, through a detailed dissection of the $2 \times k$ problem, that the basic ingredients of an efficient numerical algorithm for permutational inference comprise of, recursive generation of the distribution of the test statistic, good data structures for storing intermediate distributions through all stages of the recursion and the use of integer programming to generate a truncated distribution. The network paradigm is a useful aid for carrying out these steps. In particular, forward processing of the network is a general way to conceptualize and implement complicated recursions, whereas backward induction on the network is a general way to solve the integer programming problem.

Comment

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Professor Agresti must be congratulated for this long-awaited review of the principal issues and methodology surrounding exact inference in contingency tables. Since Fisher proposed his exact method of analysis for the 2×2 table in 1934, the amount of literature produced on the subject and the resulting debates have reached immeasurable proportions. (Yes, this pun intended!) Whether dealing with the accuracy of various asymptotic techniques in small sample situations, the diverse possible factors of correction for continuity, or the conditional, unconditional and Bayesian alternatives, the ensuing research has definitely contributed to our increased knowledge of the situation and has motivated imaginative developments in computing algorithms. Professor Agresti pre-

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