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Martin and Yohai provide an interesting study on the effect of atypical observations on the behavior of estimators in time series. The influence functional given by Definition 4.2 is the infinitesimal asymptotic bias in a one-parameter family of contaminations of a given model. The bias was also the starting point of my own paper (1984, cf. Section 1.2), but I treated only a smaller class of estimators and I focused on different aspects. So let me explain the differences between the two approaches and discuss their advantages and disadvantages.

Heuristically the connection between ICH and IF is as follows. ICH is the derivative in all directions, i.e., the gradient of $\mathbf{T}$. Hence by the chain rule of differential calculus one gets, formally,

$$\mathrm{IF} = \frac{d}{d\gamma}\mathbf{T}(\mu_y^\gamma) = < \mathrm{grad}\,\mathbf{T}, \frac{d}{d\gamma}\mu_y^\gamma > = \int \mathrm{ICH}(\mathbf{y}_1)\frac{d}{d\gamma}\mu_y(d\mathbf{y}_1).$$

If $\mathbf{T}$ depends only on the m-dimensional marginal, we can find $(d/d\gamma)\mu_y^\gamma$ in the model (2.4) by the following argument. Ignoring terms of order $o(\gamma)$, there is at most one block of outliers intersecting with $(1, 0, \dots, 2 - m)$, and the initial point of this block is distributed uniformly over $(1, 0, \dots, 3 - m - k)$. To me, the most important theoretical contribution of Martin and Yohai is Theorem 4.2 where they show that the same result also holds for $m = \infty$, at least if $\tilde{\psi}$ depends only weakly on values far away. Since the uniform distribution on all integers is not finite, a bounded $\tilde{\psi}$ is not sufficient for the boundedness of $(d/d\gamma)\mathbf{T}(\mu_y^\gamma)$.

Some of the arguments in the proof of Theorem 4.2 involve the specific contamination model while others are valid more generally. Since the latter may be useful in other situations, I propose to split it in the following way.

THEOREM 4.1′. *Let* $\mathbf{T}$ *be a* $\tilde{\psi}$ *estimate with* $\mathbf{t}_0 = \mathbf{T}(\mu_x)$ *and put* $\mathbf{m}(\gamma, \mathbf{t}) = E[\tilde{\psi}(\mathbf{y}_1^\gamma, \mathbf{t})]$. *If*

(a′) $\mathbf{T}(\mu_y^\gamma) - \mathbf{t}_0 = O(\gamma)$,

(b′) $\mathbf{m}(0, \mathbf{t})$ *is differentiable at* $\mathbf{t} = \mathbf{t}_0$ *and the derivative* C *is nonsingular*,

(c′) $\mathbf{b}(\mathbf{t}) = \lim(\mathbf{m}(\gamma, \mathbf{t}) - \mathbf{m}(0, \mathbf{t}))/\gamma$ *exists and the convergence is uniform for* $|\mathbf{t} - \mathbf{t}_0| \le \varepsilon_0$,