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CORRECTION TO

A GLIVENKO-CANTELLI THEOREM AND STRONG LAWS OF LARGE NUMBERS FOR FUNCTIONS OF ORDER STATISTICS

BY JON A. WELLNER

University of Rochester

Professors Peter Gaenssler and Winfried Stute have kindly brought to my attention an error in the proof of (B) of Theorem 1 of the above-mentioned paper (*Ann. Statist.* **5** (1977) 473–480). The proof given there is valid under the additional assumption that h is concave or convex; or if $h \geqq aI$ for some $a > 0$. It is easily seen however that there exist nonnegative, nondecreasing continuous functions h which satisfy

$$\liminf_{t\to 0} h(t)/t = 0 \qquad \text{and} \qquad \limsup_{t\to 0} h(t)/t = +\infty .$$

These functions are not concave or convex or bounded below by any line through the origin; hence the argument given in the first seven lines of the proof of Theorem 1 is invalid.

Fortunately, part (B) of Theorem 1 is true as stated (without an additional convexity or concavity or boundedness assumption as discussed above). Furthermore, (C) if $\int_0^1 (1/h)\, dI = \infty$ then

$$\limsup_{n\to\infty} \rho_h(\Gamma_n, I) = +\infty \quad \text{w.p. } 1 .$$

The following simple proof of both (B) of Theorem 1 and (C) is due to Gaenssler and Stute.

Without loss suppose $\int_0^\varepsilon (1/h)\, dI = \infty$. Then for any positive integer r and $n \geqq N = N(r, \varepsilon)$ the sequence

$$c_n \equiv \sup\{t \leqq \varepsilon : h(t)^{-1} = 2nr\}$$

is well defined and

$$\begin{aligned}\textstyle\sum_{n=n_0}^\infty c_n &= (2r)^{-1} \textstyle\sum_{n=n_0}^\infty c_n(2(n+1)r - 2nr) \\ &\geqq (2r)^{-1} \textstyle\int_0^\varepsilon (1/h)\, dI - \text{constant} = \infty .\end{aligned}$$

Thus Borel–Cantelli implies that $P(\xi_n \leqq c_n \text{ i.o.}) = 1$; and consequently $P(\xi_{n1} \leqq c_n \text{ i.o.}) = 1$ also. Since $1/h$ is continuous and nonincreasing on $(0, \varepsilon)$, this implies that

$$\rho_h(\Gamma_n, 0) = \sup_{0<t\leqq 1} (\Gamma_n(t)/h(t)) \geqq (nh(\xi_{n1}))^{-1} \geqq (nh(c_n))^{-1} = 2r$$

infinitely often w.p. 1, which yields (B) of Theorem 1 as claimed. Similarly,

$$\rho_h(\Gamma_n, I) \geqq (2nh(\xi_{n1}))^{-1} \geqq (2nh(c_n))^{-1} = r$$

infinitely often w.p. 1, which proves (C).

It should be noted that (C) together with (A) of Theorem 1 of the paper imply that finiteness of $\int_0^1 (1/h)\, dI$ is both necessary and sufficient for the weighted Glivenko–Cantelli theorem for Γ_n.