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ADDENDUM

ON THE SUPPORTS OF MEASURE-VALUED CRITICAL BRANCHING BROWNIAN MOTION

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In the paragraph preceding the statement of Theorem 3 in the introductory section of the article, a discussion of the almost-sure behaviour in the two-dimensional case is conspicuously absent. The result which fills this gap is the following one: If $\varnothing \neq G \subset \mathbb{R}^2$ is bounded and open and $X_0 = \lambda$ (Lebesgue measure), then with probability 1, $X_t(G) \nrightarrow 0$ as $t \to \infty$. Indeed if $\lim_{t \to \infty} X_t(G) = 0$ on a set Ω_0 of positive probability, then $\lim_{t \to \infty} t^{-1} \int_0^t X_s(G)\, ds = 0$ on Ω_0 as well. This however contradicts Theorem 2 of Iscoe (1986) which implies that $t^{-1} \int_0^t X_s(G)\, ds$ converges weakly, as $t \to \infty$, to $\xi \cdot \lambda(G)$ where ξ is a *strictly* positive random variable.

The corresponding result for the classical critical binary branching Brownian motion (in which all particles have the same mass) was obtained in Sawyer and Fleischman (1979) by different methods.

Thus combining the result above with ours and those of Dawson (1977), in the case $X_0 = \lambda$, we have the following simplified summary for the dependence of the behaviour of $X_t(G)$ as $t \to \infty$ for a fixed nonempty, bounded, open $G \subset \mathbb{R}^d$:

1. $d = 1$, $\exists$ finite r.v. τ such that a.s. $\forall\ t \geq \tau$, $X_t(G) = 0$.
2. $d = 2$, $\lim_{t \to \infty} X_t(G) = 0$ in probability, but w.p. 1, $X_t(G) \nrightarrow 0$, as $t \to \infty$.
3. $d \geq 3$, $\exists$ nontrivial random measure X_∞ such that $\lim_{t \to \infty} X_t(G) = X_\infty(G)$ weakly; $EX_\infty(G) = \lambda(G)$.

Finally, there is an omission in the third-to-last sentence in the proof of Proposition 4.2. That sentence should read: "Returning to (4.6), ... $r > \varepsilon$ for $d \geq 5$, so that ... $\theta \to +\infty$ (for all d since $c_\theta = 0$ for $d \leq 4$)."

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