The ratio of the mean frequency to the number of trials is therefore the probability itself. When p is small the mean error differs little from the square root $\sqrt{Np}$ of the mean frequency; and if p is nearly $= 1$, the mean error of the opposite event is nearly equal to $\sqrt{Nq}$. When the probability, p, is nearly equal to $\frac{1}{2}$, the mean error will be about $\frac{1}{2}\sqrt{N}$.

The law of error is not strictly typical, although the rational function of the r^{th} degree in $\lambda_r(m)$ vanishes for r different values of p between 0 and 1, the limits included, so that the deviation from the typical form must, on the whole, be small. If, however, we consider the relative magnitude of the higher half-invariants as compared with the powers of the mean error

$$\left.\begin{aligned} \lambda_3(m) \cdot (\lambda_2(m))^{-\frac{3}{2}} &= \frac{q-p}{\sqrt{Npq}} \\ \text{and} \qquad \lambda_4(m) \cdot (\lambda_2(m))^{-2} &= \frac{q^2 - 4pq + p^2}{Npq} \end{aligned}\right\} \tag{125}$$

the occurence of Npq in the denominators of the abridged fractions shows, not only that great numbers of repetitions, here as always, cause an approximation to the typical form, but also that, in contrast to this, the law of error in the cases of certainty and impossibility, when $q = 0$ and $p = 0$, becomes skew and deviates from the typical in an infinitely high degree, while at the same time the square of the mean errors becomes $= 0$. This remarkable property is still traceable in the cases in which the probability is either very small or very nearly equal to 1. In a hundred trials with the probability $= 99\frac{1}{2}$ per ct. the mean error will be about $= \sqrt{\frac{1}{2}}$. Errors beyond the mean frequency $99\frac{1}{2}$ cannot exceed $\frac{1}{2}$, and are therefore less than the mean error. The great diminishing errors must therefore be more frequent than in typical cases, and frequencies of 97 or 96 will not be rare in the case under consideration, though hey must be fully counter-balanced by numerous cases of 100 per ct. The law of error is consequently skew in a perceptible degree. In applications of adjustment to problems of probability, it is, from this reason, frequently necessary to reject extreme probabilities.

XV. THE FORMAL THEORY OF PROBABILITY.

§ 67. The formal theory of probability teaches us how to determine probabilities that depend upon other probabilities, which are supposed to be given. Of course, there are no mathematical rules specially applicable to computations that deal with probabilities, and there are many computations with probabilities which do not fall under the theory of probability, for instance, adjustments of probabilities. But in view of the direct application