The deviations are evidently too large ($o-u$ is $+2{\cdot}4$, $-1{\cdot}4$, $-2{\cdot}2$, $-1{\cdot}0$, $+2{\cdot}2$) to be due to the use of round numbers; the sum of the squares is also

$$220{\cdot}8 \text{ instead of } 3 \pm \sqrt{6},$$

consequently, no doubt, an over-adjustment.

The special adjustment of the second degree, $\Delta^4 = 0$, $\nabla\Delta^3 = 0$, and $\Delta^2 = 2$, gives for u_i and its differences:

$$\begin{array}{ccccccccc} 11{\cdot}6 & & 19{\cdot}4 & & 29{\cdot}2 & & 41{\cdot}0 & & 54{\cdot}8 \\ & 7{\cdot}8 & & 9{\cdot}8 & & 11{\cdot}8 & & 13{\cdot}8 & \end{array}$$

The deviations $o-u =$ $\cdot\, 0{\cdot}4$, $-0{\cdot}4$, $-0{\cdot}2$, $0{\cdot}0$, $+0{\cdot}2$

nowhere reach $\frac{1}{2}$, and may consequently be due to the use of round numbers; the sum of the squares

$$4{\cdot}8 \text{ instead of } 3 \pm \sqrt{6}$$

also agrees very well. Indeed, a constant subtraction of 0·04 from u_i would lead to $(3{\cdot}4)^2$, $(4{\cdot}4)^2$, $(5{\cdot}4)^2$, $(6{\cdot}4)^2$, and $(7{\cdot}4)^2$, from which the example is taken.

Example 3. Between 4 points on a straight line the 6 distances

$$\begin{array}{ccc} o_{12}, & o_{13}, & o_{14} \\ & o_{23}, & o_{24} \\ & & o_{34}\,. \end{array}$$

are measured with equal exactness without bonds. By adjustment we find for instance

$$u_{12} = \tfrac{1}{2} o_{12} + \tfrac{1}{4}(o_{13} - o_{23}) + \tfrac{1}{4}(o_{14} - o_{24});$$

we notice that every scale $= \frac{1}{2}$. It is recommended actually to work the example by a millimeter scale, which is displaced after the measurement of each distance in order to avoid bonds.

XII. ADJUSTMENT BY ELEMENTS.

§ 51. Though every problem in adjustment may be solved in both ways, by correlates as well as by elements, the difficulty in so doing is often very different. The most frequent cases, where the number of equations of condition is large, are best suited for adjustment by elements, and this is therefore employed far oftener than adjustment by correlates.

The adjustment by elements requires the theory in such a form that *each observation is represented by one equation* which expresses the mean value $\lambda_1(o)$ explicitely as linear functions of unknown values, the "*elements*", x, y, ... z: