

On the other hand, if, in a table arranged according to the magnitude of the values, we select a single middle value, preceded and followed by nearly equal numbers of values, we shall get a quantity which is very well fitted to represent the whole series of repetitions.

If, while we are thus counting the results arranged according to their magnitude, we also take note of these two values with which we respectively (a) leave the first sixth part of the total number, and (b) enter upon the last sixth part (more exactly we ought to say 16 per ct.), we may consider these two as indicating the limits between great and small deviations. If we state these two values along with the middle one above referred to, we give a serviceable expression for the law of errors, in a way which is very convenient, and although rough, is not to be despised. Why we ought to select just the middle value and the two sixth-part values for this purpose, will appear from the following chapters.

IV. CURVES OF ERRORS.

§ 12. Curves of actual errors of repeated observations, each of which we must be able to express by one real number, are generally constructed as follows. On a straight line as the axis of abscissae, we mark off points corresponding to the observed numerical quantities, and at each of these points we draw an ordinate, proportional to the number of the repetitions which gave the result indicated by the abscissa. We then with a free hand draw the curve of errors through the ends of the ordinates, making it as smooth and regular as possible. For quantities and their corresponding abscissae which, from the nature of the case, *might* have appeared, but do not really appear, among the repetitions, the ordinate will be $= 0$, or the point of the curve falls on the axis of abscissae. Where this case occurs very frequently, the form of the curves of errors becomes very tortuous, almost discontinuous. If the observation is essentially bound to discontinuous numbers, for instance to integers, this cannot be helped.

§ 13. If the observation is either of necessity or arbitrarily, in spite of some inevitable loss of accuracy, made in round numbers, so that it gives a lower and a higher limit for each observation, a somewhat different construction of the curve of errors ought to be applied, viz. such a one, that the area included between the curve of error, the axis of abscissae, and the ordinates of the limits, is proportional to the frequency of repetitions within these limits. But in this way the curve of errors may depend very much on the degree of accuracy involved in the use of round numbers. This construction of areas can be made by laying down rectangles between the bounding ordinates, or still better, trapezoids with their free sides approximately parallel to the tangents of the curve. If the