RANK CORRELATION AND TESTS OF SIGNIFICANCE INVOLVING NO ASSUMPTION OF NORMALITY*, †

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1. Dependence of Tests of Significance on Normality

The powerful tests of significance, largely the work of R. A. Fisher, which have been revolutionizing statistical theory and practice, are in the main based on the assumption of a normal distribution in a hypothetical population from which the observations are a random sample. The nature and extent of the errors likely to result from the application of a test of significance assuming normality, where normality does not really exist, have been the subject of investigations both experimental and mathematical,[1] which however have not produced satisfactory substitutes for Fisher's methods. A false assumption of normality does not usually give rise to serious errors in the interpretation of simple means, since the distribution of a mean of any considerable number of cases is very nearly normal, no matter what the nature of the parent population, so long as it does not fall within a certain class having infinite range, and including the Cauchy distribution. The sampling distributions of second-order statistics are however more seriously disturbed by lack of normality, as is evident from their standard errors. For example the variance $(\mu_4 - \mu_2^2)/n$ of sample variances is much affected if μ_4/μ_2^2 differs considerably, as it often does, from the value 3 which it takes for a normal distribution. Likewise the approximate variance of the correlation coefficient,

$$\sigma_r^2 = \frac{1}{n\mu_{20}\mu_{02}}\left\{\mu_{22} + \frac{\mu_{40}\mu_{11}^2}{4\mu_{20}^2} + \frac{\mu_{04}\mu_{11}^2}{4\mu_{02}^2} - \frac{\mu_{31}\mu_{11}}{\mu_{20}} - \frac{\mu_{13}\mu_{11}}{\mu_{02}} + \frac{\mu_{22}\mu_{11}^2}{2\mu_{20}\mu_{02}}\right\},$$

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[1] J. L. Carlson, *A Study of the Distribution of Means Estimated from Small Samples by the Method of Maximum Likelihood for Pearson's Type II Curve*, Unpublished M. A. Thesis, Leland Stanford Junior University, 1931.

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