$$\varphi(m', \lambda^2) \leq F \leq \varphi(m'', \lambda^2). \tag{9}$$

Denote by $\lambda_1'^2, \lambda_1''^2, \lambda_2'^2, \lambda_2''^2$ the roots in λ^2 of the following equations respectively:

$$\varphi(m', \lambda^2) = F_2 ;$$

$$\varphi(m'', \lambda^2) = F_2 ;$$

$$\varphi(m', \lambda^2) = F_1 ; \qquad \varphi(m'', \lambda^2) = F_1 .$$

Since F is monotonically decreasing with increasing λ^2, on account of (7), (8), and (9) we obviously have

$$\lambda_1'^2 \leq \lambda_1^2 \leq \lambda_1''^2$$

and

$$\lambda_2'^2 \leq \lambda_2^2 \leq \lambda_2''^2.$$

The above inequalities give us the required limits.

COLUMBIA UNIVERSITY,
NEW YORK, N. Y.

THE DISTRIBUTION OF QUADRATIC FORMS IN NON-CENTRAL NORMAL RANDOM VARIABLES

BY WILLIAM G. MADOW[1]

The following theorem is the algebraic basis of the theorem of R. A. Fisher and W. G. Cochran which states necessary and sufficient conditions that a set of quadratic forms in normally and independently distributed random variables should themselves be independently distributed in χ^2-distributions.[2]

THEOREM I. *If the real quadratic forms* $q_1, \cdots, q_m$, *in* $x_1, \cdots, x_n$, *are such that*

$$\sum_\gamma q_\gamma = \sum_\nu x_\nu^2, \tag{1}$$

and if the rank of q_γ *is* n_γ, *then a necessary and sufficient condition that*

$$q_\gamma = \sum_\alpha z_\alpha^2, \tag{2}$$

---

[1] The letters i, j, μ, ν will assume all integral values from 1 through n, the letter γ will assume all integral values from 1 through m, $(n \geq m)$, the letter α will assume all integral values from $n_1 + \cdots + n_{\gamma-1} + 1$ through $n_1 + \cdots + n_\gamma$, $(n_0 = 0,\ n_1 + \cdots + n_m = n')$, the letters β, β' will assume all integral values from 1 through n', and the letters r, s will assume all integral values from 1 through $n - 1$.

[2] The references are: W. G. Cochran, "The Distribution of Quadratic Forms in a Normal System, with Applications to the Analysis of Covariance," *Proc. Camb. Phil. Soc.*, Vol. 30 (1934), pp. 178–191, and R. A. Fisher, "Applications of 'Student's' Distribution," *Metron*, Vol. 5 (1926), pp. 90–104.