SAMPLES FROM TWO BIVARIATE NORMAL POPULATIONS[1]

By Chung Tsi Hsu

Columbia University

1. **Introduction.** In multivariate analysis involving p variates, or in analysis of variance of m samples from univariate populations, we are often interested in the hypothesis of the equality of variances; viz., that

$$\sigma_1 = \sigma_2 = \cdots = \sigma_p, \qquad \text{in the case of } p \text{ variates;}$$

or

$$\sigma_1 = \sigma_2 = \cdots = \sigma_m, \qquad \text{in the case of } m \text{ samples.}$$

As a matter of fact, it seldom occurs that these hypotheses are true, but the ratio between the variances might be known.

Hotelling [5] has suggested that if

$$\sigma_1^2/k_1 = \sigma_2^2/k_2 = \cdots = \sigma_m^2/k_m = \sigma^2,$$

where the k's are known constants, we can apply the transformation

$$\begin{aligned} x_1' &= w_1 x_1, \\ x_2' &= w_2 x_2, \\ &\cdots\cdots\cdots \\ x_m' &= w_m x_m, \end{aligned}$$

where

$$w\sqrt{k_1} = w_2\sqrt{k_2} = \cdots = w_m\sqrt{k_m} = 1,$$

so that after transformation the variances become equal, i.e.,

$$\sigma_1' = \sigma_2' = \cdots = \sigma_m',$$

and the required analysis can be carried out. This method is similarly applicable in the multivariate case.

In a previous paper [7], I developed a series of hypotheses concerning samples from a bivariate normal population under the assumption that

$$\sigma_1 = \sigma_2.$$

In case $\sigma_1^2/k_1 = \sigma_2^2/k_2$, where k_1 and k_2 are two distinct known constants, similar results may be obtained by the use of the transformation $x_1' = w_1x_1$; $x_2' = w_2x_2$; where $w_1\sqrt{k_1} = w_2\sqrt{k_2} = 1$.

[1] Presented to the American Mathematical Society at Washington, D. C., May 3, 1941.