one infers that the probability is greater than .95 that for a sample showing such a large deviation from the mean ($u/\sqrt{n} = 4, n = 4$) *all* the constituent elements will have deviations on the same side of the population mean. Thus if all the elements of the sample investigated are found to have deviations on the same side of the population mean, this could *not* be construed as *additional evidence* that the sample indicated an abnormal condition.

This conclusion is weaker than the facts of the example warrant, since it is based upon the *integral* of $F_n(u)$ from u' to infinity. Unfortunately the author does not have data available on the rate of convergence of these integrals.

NOTE ON A MATRIC THEOREM OF A. T. CRAIG

By Harold Hotelling

Columbia University

An extremely elegant theorem given recently by A. T. Craig[1] and applied by him to establish a further theorem on independent χ^2 distributions may be stated as follows:

If A and B are the symmetric matrices of two homogeneous quadratic forms in n variates which are normally and independently distributed with zero means and unit variances, a necessary and sufficient condition for the independence in probability of these two forms is that $AB = 0$.

The proof given that the condition is sufficient is adequate, but Craig's treatment of its necessity consists essentially in its assertion. In view of the growing interest in such quadratic forms, for example in connection with serial correlation, the neatness of this theorem is likely to lead to a wide usefulness. It therefore seems worth while to give a complete proof of the necessity condition.

The form with matrix A is denoted by Q_1 and that with matrix B by Q_2. The characteristic functions, if defined as $Ee^{\frac{1}{2}\lambda Q_1}$ and $Ee^{\frac{1}{2}\mu Q_2}$, are respectively the reciprocals of the square roots of the determinants of the matrices $1 - \lambda A$ and $1 - \mu B$, while the characteristic function for Q_1 and Q_2 together, $Ee^{\frac{1}{2}(\lambda Q_1 + \mu Q_2)}$, is the reciprocal of the square root of the determinant of $1 - \lambda A - \mu B$. A necessary and sufficient condition for independence is therefore that

$$| 1 - \lambda A | \cdot | 1 - \mu B | \equiv | 1 - \lambda A - \mu B |$$

shall hold identically for all values of λ and μ. Since the determinant of the product of two matrices is the product of their determinants, the left member is the same as

$$| 1 - \lambda A - \mu B + \lambda\mu AB | .$$

From this it is immediately obvious that $AB = 0$ implies the independence of the two forms. The converse will now be proved.

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[1] "Note on the independence of certain quadratic forms," *Annals of Math. Stat.*, Vol. 14 (1943), pp. 195–197.